MESONIC INTERACTIONS IN A BROKEN SU(3) X SU(3) EFFECTIVE LAGRANGIAN MODEL

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CERTIFICATE

Certified that the work reported in the thesis entitled 'Mesonic Interactions in a Broken SU(3) X SU(3) Effective Lagrangian Model' by Ashok Kumar Bhargava has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

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SYNOPSIS

In this work we report on some investigations on an effective Lagrangian model which is a generalisation of the σ model to chiral SU(3) X SU(3) and is further extended to include spin one fields introduced as gauge fields. The SU(3) X SU(3) currents are either conserved or satisfy partial conservation equations. The vacuum is invariant only under a subgroup of chiral SU(3) X SU(3) namely SU(2) X U(1) corresponding to isospin and hypercharge symmetry. All the calculations are done in the tree graph approximation.

The spin zero fields are assigned the $(3, 3^*) + (3^*, 3)$ representation of SU(3) X SU(3) and parity. The spin one fields, by virtue of being gauge fields, belong to the regular representation (8, 1) + (1, 8). Apart from the vacuum breaking, one has to include two other types of symmetry breaking as well. A common mass term for gauge fields invariant only under coordinate independent SU(3) X SU(3) transformations, is included to ensure nonzero mass for gauge fields corresponding to conserved currents. Another term is a linear function of scalar fields transforming as $(3, 3^*) + (3^*, 3)$ representation of SU(3) X SU(3) which breaks the symmetry intrinsically to SU(2) X U(1). It is there to ensure partial conservation of currents and for avoiding the unpleasant consequences of Goldstone theorem.

The thesis is divided into six chapters. The first chapter is in the form of a general introduction. In the second chapter the necessary mathematical background for constructing a Lagrangian invariant under co-ordinate dependent SU(3) X SU(3) symmetry transformations is developed. A brief account of spontaneous breaking and Yang-Mills approach is given. The covariant derivatives and invariants for spin zero and spin one fields are obtained.

The basic Lagrangian is written down in the third chapter. One would like to stick to minimal couplings of the gauge fields; however, it turns out that in order to describe particle masses and decay of axial vector mesons correctly, it is necessary to include some nonminimal couplings as well. The vacuum breaking is introduced by giving nonzero vacuum expectation values to the appropriate scalar fields. After taking account of field mixings and renormalizations the physical spin one and spin zero fields are defined. Most of the parameters of the model are determined from some of the masses and remaining masses are predicted. The predicted masses including those of scalar mesons are close to the masses of possible experimental candidates.

The fourth chapter deals with strong processes involving mesons. Widths of two particle decay modes of vector, axial vector and scalar mesons and three particle decay modes of

axial vector and pseudoscalar mesons are calculated. Finally the elastic scattering of two pseudoscalar mesons is considered; in particular we calculate π-π (s-and p-wave), K-π (s-wave) and K-K (s-wave) scattering lengths and effective ranges.

The fifth chapter is devoted to the study of vector and axial vector currents. The expressions for vector and axial vector currents are obtained which along with coupling constants obtained earlier are used in calculating decay constants of mesons, K_{13} and K_{14} form factors.

In the sixth and last chapter, we discuss some possible improvements of the model namely inclusion of ninth vector and axial vector mesons. We find that if these are included one has to introduce a large number of additional parameters which reduces the predictive power of the model considerably. Then we discuss the situation where the explicit symmetry breaking linear term is absent. This provides an illustration of Goldstone theorem and some of the results of Higgs and Kibble work. Finally, a limit of the model is discussed in which masses of chiral partners of the Goldstone bosons becomes infinite. The predictions in this limit are compared with those of current algebra and nonlinear effective Lagrangians.

Finally, there is an appendix in which all the coupling constants used in the calculations are listed.

CHAPTER I

INTRODUCTION

Gell-Mann's current algebra hypothesis[1] has led to considerable understanding of weak and strong interactions of the hadrons. According to this hypothesis the fourth component of vector and axial vector currents, which take part in weak interactions, satisfy, at equal times, SU(3) X SU(3) algebra even though the strong interaction Lagrangian may not be SU(3) X SU(3) symmetric. In order to calculate matrix elements of currents one has to supplement current algebra with more specific assumptions. The two main assumptions commonly made are:

(i) the axial vector currents and some of the vector currents (strange vector current) are partially conserved.[2] By partial conservation of currents we mean that the matrix elements of the divergence of the current are dominated by appropriate spin zero meson pole. In a field theoretical model this is implemented by requiring currents to satisfy identities like,

$$\begin{aligned} \partial_\mu J^V_\mu(x) &= \text{const. } s(x) , \\ \partial_\mu J^A_\mu(x) &= \text{const. } p(x) , \end{aligned} \qquad (1.1)$$

where s and p are appropriate scalar and pseudoscalar fields respectively.

(ii) the matrix elements of the currents are dominated by the spin one meson poles.[3] This is commonly ensured by replacing currents with corresponding spin one interpolating fields.[4]

Weinberg[5] showed that the current algebra results for π-π and N-π scattering lengths can be easily derived from an effective Lagrangian obtained from the SU(2) X SU(2) σ model[6] after performing a canonical transformation on the nucleon field and taking the limit $M_{\sigma}^2 \rightarrow \infty$. Soon after it was noted by several authors[6,7] that the above mentioned assumptions can be economically incorporated in suitably constructed effective Lagrangians with which calculations are to be done in the tree graph approximations. Furthermore in an effective Lagrangian model one can test the validity of more specific assumptions as well; one is sure of the consistency of the model and can see the interdependence of the various processes which is missing in the usual treatment of individual processes.

The first step in constructing a Lagrangian is to assign a suitable representation of the group to the particles to be considered. The spin zero mesons are usually assigned to $(3, 3^*) + (3^*, 3)$ representation of chiral SU(3) X SU(3) and parity. In order to fill this representation one needs nine

pseudoscalar and nine scalar particle. In the year 1967, since there was no convincing evidence for the existence of scalar particles, ways were found to construct SU(3) X SU(3) invariant Lagrangians without these. It was observed[7] that this is possible if one assumes that some of the experimentally observed fields transform nonlinearly under SU(3) X SU(3) transformations. From these nonlinearly transforming fields linearly transforming functions were constructed which were subsequently used in constructing Lagrangians, commonly known as nonlinear Lagrangians. If these scalar particles are observed - there is growing evidence in favour of these - then either one has to abandon or greatly modify this approach. On the other hand one assumes that fields transform linearly, all of them have equal status, then the linear Lagrangians so constructed will contain more information than the nonlinear Lagrangians. In this work, we shall construct a linear Lagrangian where all the fields are treated on the equal footing.

As far as the question of including spin one meson goes, one of the possibility is to assign some suitable representation of the group to these and to write down invarient coupling terms. However, when this is done the resulting Lagrangian has too many parameters, so this is not an attractive possibility. Another possibility is to follow the Yang-Mills[8] type approach. In this approach one demands invariance under coordinate dependent group transformations. The spin one

fields are identified with gauge fields which are introduced to maintain invariance under the extended symmetry. The gauge fields couple universally with other fields and there is only one coupling constant.

In nature SU(3) X SU(3) cannot be an exact symmetry; already the subgroup SU(3) is a broken symmetry. Moreover, most of the SU(3) X SU(3) currents are known to be not conserved. Therefore, one has to introduce symmetry breaking. The possibility that vacuum may not be symmetric under the full group but is only invariant under a subgroup is attractive in itself. When this is so we say that the symmetry is broken spontaneously. In these theories, however, due to Goldstone theorem,[9,10] there must exist zero mass particles, Goldstone bosons (Goldstones), contrary to the experiments. However, in gauge theories when symmetry is broken spontaneously the Goldstones become the longitudinal modes of gauge fields and are removed from the theory.[11,12] This way the gauge fields also become massive. However, only those gauge field acquire mass which correspond to the broken components of the symmetry.[12] In case of our symmetry group SU(3) X SU(3), which is broken spontaneously to SU(2) X U(1), then the octet of pseudoscalars and the scalar kaon appear as Goldstones. In the completely gauge invariant theory these get combined with gauge fields and are eliminated from the model which is disastrous. This complete elimination can be prevented if we add

a common mass term for gauge fields, which is invariant under coordinate independent group transformations only. It also ensures nonzero mass to the gauge fields coupled to the conserved currents. Although the complete elimination of Goldstones is prevented but they are still massless. Further more all the currents are conserved whereas we want some of them to be partially conserved. Both of these difficulties are overcome if one adds a symmetry breaking term transforming as some representation of the symmetry group. In SU(3) X SU(3) the most economical explicit symmetry breaking[13] turns out to be a linear function of scalar fields transforming as $(3,3^*)+(3^*,3)$ representation of SU(3) X SU(3).

The Lagrangian which can be written down from these requirements is not unique. This is due to the fact that many invariants involving fields and their derivatives can be constructed. However, one naturally likes to write the simplest possible Lagrangian unless forced to do otherwise. Although one would like to stick to minimal couplings of vector mesons obtained by replacing ordinary derivatives of fields by covariant derivatives, it turns out that in order to give satisfactory description of masses of particles and their decays, one has to include some nonminimal coupling terms.

Since we started this work some papers reporting similar investigations have appeared in the literature.[14-16] Although there is some overlap with these works, the present work[17] is

distinguished from the fact that a variety of dynamical quantities like particle masses, couplings, decay rates, scattering lengths and form factors have been calculated from the same effective Lagrangian and numerical details have been worked out.

The plan of the thesis is as follows. In the second chapter we collect necessary mathematical techniques used in constructing invariants under coordinate dependent SU(3) X SU(3) transformations. After giving a brief account of spontaneous breaking and Yang-Mills approach the covariant derivatives and invariants for spin zero and spin one fields are constructed. The basic Lagrangian for nonets of spin zero mesons and octets of spin one mesons is written down in third chapter. Vacuum breaking is introduced by giving nonzero vacuum expectation values to the appropriate scalar fields. The field mixings are removed and renormalized spin one and spin zero fields are identified. Most of the parameters of the model are determined from some of the masses and remaining masses including those of the scalar mesons are predicted. The fourth chapter deals with strong processes involving mesons. Widths of two particle decay modes of vector, axial vector and scalar mesons and three particle decay modes of axial vector and pseudoscalar mesons are calculated. Finally, the elastic scattering of π-π, K-π and K-K are considered. The fifth chapter is devoted to the study of vector and axial vector currents which are subsequently used in

calculating decay constants of mesons, K_{l3} and K_{l4} form factors. In the sixth chapter we discuss some possible improvements of the model namely inclusion of ninth vector and axial vector mesons. Then we discuss the situation where the explicit symmetry breaking linear term is absent. A limit of the model is discussed in which masses of chiral partners of the Goldstones become infinite. Its predictions are compared with those of current algebra and nonlinear effective Lagrangians.

Finally, there is an appendix in which all the coupling constants used in the calculations are listed.

CHAPTER II

CHIRAL SU(3) X SU(3) AND SYMMETRY BREAKING

This chapter is devoted to the construction of invariants under the coordinate dependent SU(3) X SU(3) transformations and to the form of symmetry breaking. In Section 1, after introducing group SU(3) X SU(3) as a symmetry of vector and axial vector currents we construct the invariants of fields belonging to $(3, 3^*) + (3^*, 3)$ and $(8, 1) + (1, 8)$ representation of SU(3) X SU(3). The generalization of symmetry transformations considered in Section 1 to space time dependent group parameters is considered in Section 2. The requirement that the Lagrangian should be invariant under the generalised symmetry transformations leads to the introduction of gauge fields. Section 3 contains the discussion of the consequences of assumption that vacuum is not invariant under the symmetry transformations. When this happens massless particles, Goldstones, appear in the theory. We discuss how to cure masslessness of Goldstones and at the same time of gauge fields.

2-1 The Group Chiral SU(3) X SU(3) and its Representations:

It was proposed by Gell-Mann that the vector currents, J^V_μ, which take part in the weak processes of strongly interacting particles belong to the octet representation of SU(3). The operator Q^V_k ($k = 1 \ldots 8$) defined as,

$$Q^V_k = \int d^3x \, J^V_{ok}(x), \tag{2.1}$$

are the generators of SU(3). The axial vector currents, J^A_μ, also belong to an octet representation of SU(3). The operators Q^A_k ($k = 1 \ldots 8$) defined as,

$$Q^A_k = \int d^3x \, J^A_{ok}(x), \tag{2.2}$$

are the generator of axial SU(3). These sixteen operators satisfy the following equal time commutation relations,

$$\begin{aligned} [Q^V_k, Q^V_l]_- &= i f_{klm} Q^V_m, \\ [Q^V_k, Q^A_l]_- &= i f_{klm} Q^A_m, \\ [Q^A_k, Q^A_l]_- &= i f_{klm} Q^V_m, \end{aligned} \tag{2.3}$$

where f_{klm} are real and totally antisymmetric structure constants of SU(3). We define,

$$Q^{\pm}_k = \frac{1}{2}(Q^V_k \pm Q^A_k). \tag{2.4}$$

The operators Q^+_k and Q^-_k seperately obey the commutation relations,

$$[Q^{\pm}_k, Q^{\pm}_l]_- = i f_{klm} Q^{\pm}_m, \tag{2.5}$$

while they commute with each other,

$$[\, Q_k^+ ,\; Q_l^- \,]_- \;=\; 0. \qquad (2.6)$$

Thus we are now dealing with a direct product of two SU(3) groups that is chiral $SU_+(3) \times SU_-(3)$. The unitary transformations corresponding to these $SU_+(3)$ and $SU_-(3)$ are represented by the operators,

$$U_{\pm} \;=\; \exp\,[\, -i\, \epsilon_{\pm} \,.\, Q^{\pm} \,]\;. \qquad (2.7)$$

The $\epsilon_{\pm}$ are defined as,

$$\epsilon_{\pm} \;=\; \epsilon^V \pm \epsilon^A. \qquad (2.8)$$

The dot product $\epsilon_{\pm}.Q^{\pm}$ is shorthand notation for $\sum_{k=1}^{8} \epsilon_{\pm}^k \, Q_k^{\pm}$. The operators Q_k^+ and Q_k^- are connected to each other through the parity operation,

$$P\, Q_k^{\pm}\; P^{-1} \;=\; Q_k^{\mp}\;. \qquad (2.9)$$

We shall be concerned with irreducible representations of the larger group $SU_+(3) \times SU_-(3)$ consisting of Q_k^+, Q_k^- and parity. The irreducible representations (IR) of SU(3) are denoted by the dimensionality of its representation. In SU(3) X SU(3) an IR will be given by two numbers say (m, n) where m is the dimension of the IR with respect to $SU_+(3)$ and n is the dimension of the IR with respect to $SU_-(3)$. Since parity interchanges Q_k^+ and Q_k^- an IR of SU(3) X SU(3) and parity is given by (m, n) + (n, m) for $n \neq m$.

Let us consider the transformation properties of fields under SU(3) X SU(3). A field ψ_a (a = 1 ... m) belonging to

(m, o) transforms as,

$$U_+ \psi_a U_+^{-1} = \psi_c \left[\exp\left(-i\epsilon_+ \cdot F_+\right)\right]_a^c , \qquad (2.10)$$

where F_+ forms a matrix representation of the Q^+ generators in the m dimensional representation. Since F_+ is a m $\times$ m matrix, indices c and a (c, a = 1 ... m) denote the ca-th element of the matrix exp $(-i\epsilon_+ . F_+)$. From (10) we obtain[19] for ψ^a belonging to the contragradient representation (m^*, o)

$$U_+ \psi^a U_+^{-1} = \psi^c \left[\left(\exp\left(-i\epsilon_+ . F_+\right)^{-1}\right)^T \right]_a^c$$

$$= \left[\exp\left(i\epsilon_+ . F_+\right) \right]_c^{\ a} \psi^c . \qquad (2.11)$$

The transformation law for a field $\psi_{\dot{b}}$ ($\dot{b}$ = 1 ... n) belonging to (o, n) is given by,

$$U_- \psi_{\dot{b}} U_-^{-1} = \psi_{\dot{d}} \left[\exp\left(-i\epsilon_- . F_-\right) \right]_{\dot{b}}^{\dot{d}} , \qquad (2.12)$$

where F_- forms a matrix representation of the Q^- generators in the n representation. For $\psi^{\dot{b}}$ belonging to the contragradient representation (o, n^*) the transformation law is,

$$U_- \psi^{\dot{b}} U_-^{-1} = \left[\exp\left(i\, \epsilon_- . F_-\right) \right]_{\dot{d}}^{\dot{b}} \psi^{\dot{d}} . \qquad (2.13)$$

Thus a mixed tensor $\psi_a^{\dot{b}}$ which belongs to (m^*, n) representation transforms as*

$$U \psi_{\dot{b}}^a U^{-1} = \left[\exp(i\epsilon_+ . F_+)\right]_c^a \psi_{\dot{d}}^c \left[\exp(-i\epsilon_- . F_-)\right]_{\dot{b}}^{\dot{d}} . \quad (2.14)$$

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* In rest of this section $U = U_+ U_-$.

The IR's of SU(3) X SU(3) which are of interest to us are $(3, 3^*) + (3^*, 3)$ and $(8, 1) + (1, 8)$. First consider transformations of a tensor belonging to $(3, 3^*) + (3^*, 3)$ representation. A component ba (a, b = 1 ... 3) of 3×3 matrix field M belonging to $(3, 3^*)$ representation transforms as,

$$U M_a^{\dot{b}} U^{-1} = [\exp(i\epsilon_-.\lambda/2)]_{\dot{d}}^{\dot{b}} M_c^{\dot{d}} [\exp(-i\epsilon_+.\lambda/2)]_a^c$$
$$= [\exp(i\epsilon_-.\lambda/2) M \exp(-i\epsilon_+.\lambda/2)]_a^{\dot{b}}, \quad (2.15)$$

where the Gell-Mann matrices $\lambda_i/2$ (i = 1 ... 8) forms 3-dimensional representation of the generators of SU(3). In (15) the notations $M_a^{\dot{b}} = (M)_{\dot{b}a}$ have been used. From (15) we obtain, for the transformation properties of matrix field M,

$$U M U^{-1} = \exp(i\epsilon_-.\lambda/2) M \exp(-i\epsilon_+.\lambda/2). \quad (2.16)$$

For infinitesimal ϵ, it gives,

$$\delta M = i\epsilon_-.\lambda/2 M - i M\epsilon_+.\lambda/2$$
$$= i[\epsilon^V.\lambda/2, M]_- - i[\epsilon^A.\lambda/2, M]_+ \quad (2.17)$$

In writing last line we have made use of (8). The matrix field M^+ will according to (16) transform as,

$$U M^+ U^{-1} = \exp(i\epsilon_+.\lambda/2) M^+ \exp(-i\epsilon_-.\lambda/2), \quad (2.18)$$

and therefore it belongs to $(3^*, 3)$ representation. For infinitesimal ϵ, it gives,

$$\delta M^+ = i\epsilon_+.\lambda/2\, M^+ - i\, M^+\, \epsilon_-.\lambda/2$$
$$= i \left[\epsilon^V.\lambda/2, M^+ \right]_- + i \left[\epsilon^A.\lambda/2, M^+ \right]_+ \qquad (2.19)$$

The (16) and (18) give,

$$U\, MM^+\, U^{-1} = \exp\,(i\epsilon_-.\lambda/2)\, MM^+ \exp(-i\epsilon_-.\lambda/2) \qquad (2.20)$$

Hence,

$$\mathrm{Tr}\ (MM^+)\ , \qquad (2.21)$$

is an invariant. Similarly,

$$\mathrm{Tr}\ (MM^+\ MM^+), \qquad (2.22)$$

is also an invariant. Consider the transformation of det M,

$$U \det M\, U^{-1} = \det \left[U\, M\, U^{-1} \right]$$
$$= \det \left[\exp(i\epsilon_-.\lambda/2)\, M \exp(-i\epsilon_+.\lambda/2) \right]$$
$$= \det M. \qquad (2.23)$$

Hence,

$$\det M + \det M^+, \qquad (2.24)$$

is an invariant under SU(3) X SU(3) and parity.

Now consider the transformation properties of fields belonging to (8, 1) + (1, 8) representation. The fields L_a (a = 1 ... 8) belonging to (8, 1) transform as,

$$U\, L_a\, U^{-1} = L_b \left[\exp(-i\epsilon_+.F_+) \right]_a^b\ . \qquad (2.25)$$

For infinitesimal ϵ,

$$\delta L_a = L_b\ (-i\epsilon_+.F_+)_a^b \qquad (2.26)$$

Since in regular representation,

$$(F_c)_a^b \quad = \quad - i f_{cba}, \tag{2.27}$$

we get,

$$\delta L_a \quad = \quad -L_b f_{cba} \epsilon_{+c} \quad = \quad f_{abc} \epsilon_{+c} L_b$$

or

$$\delta L \quad = \quad i [\epsilon_+ . \lambda/2, L], \tag{2.28}$$

where,

$$L = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda_a L_a. \tag{2.29}$$

Then (25) becomes,

$$U L U^{-1} = \exp(i\epsilon_+ . \lambda/2) L \exp(-i\epsilon_+ . \lambda/2). \tag{2.30}$$

Similarly, a matrix field R,

$$R = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda_a R_a, \tag{2.31}$$

belonging to (1, 8) transforms as,

$$U R U^{-1} = \exp(i\epsilon_- . \lambda/2) R \exp(-i\epsilon_- . \lambda/2). \tag{2.32}$$

From (30) and (32) one see that the term,

$$\mathrm{Tr} \quad (LL + RR), \tag{2.33}$$

is an invariant of the group.

2-2 Generalized Gauge Transformations:

The symmetry transformations considered so far can be generalized so that the group parameters $\epsilon(x)$ are functions of space time. Under these generalised transformations, the group invariants which involve derivatives of the field are no longer

invariant. However, invariance can be maintained if one introduces vector fields, usually known as gauge fields, with appropriate transformation properties so that extra terms from the transformation on the derivatives of the matter field are cancelled by the corresponding terms coming from the transformation of vector fields. In this way, the generalisation of the group parameters from constants to function of space time can be taken as the principle generating the vector fields. For example, in quantum electrodynamics such a generalisation leads to the introduction of the electromagnetic field.

Under the gauge transformation of first kind a charge carrying field ψ transforms as,

$$\psi'(x) = \exp\left[-i\epsilon(x) \right] \psi(x) \tag{2.34}$$

and

$$\partial_\mu \psi'(x) = \exp\left[-i\epsilon(x) \right] (\partial_\mu - i\partial_\mu \epsilon(x))\psi(x). \tag{2.35}$$

In order to maintain invariance of kinetic energy term $-1/2\,(\partial_\mu \psi)^2$, the $\partial_\mu \epsilon(x)$ term must somehow be cancelled. To achieve this one introduces a vector field A_μ, the electromagnetic field, which undergoes transformation of the second kind,

$$A_\mu \rightarrow A_\mu - \frac{1}{e}\, \partial_\mu \epsilon(x), \tag{2.36}$$

where e is the electric charge. If in the Lagrangian $\partial_\mu \psi$ is replaced by the co-variant derivative,

$$D_\mu \psi = (\partial_\mu - ie\, A_\mu)\, \psi(x), \tag{2.37}$$

then clearly the resulting Lagrangian would be invariant under the transformations (34) and (36). The kinetic energy term, for the electromagnetic field, which is invariant under (36) is,

$$- \frac{1}{4} F_{\mu\nu} F_{\mu\nu} , \tag{2.38}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

It is to be noted that bare mass of A_μ field is zero because term like $A_\mu A_\mu$ is not invariant under the transformation(36).

Yang and Mills[8] applied this procedure to the group of isospin transformations of a nucleon field ψ. As in the electromagnetic case, in order to preserve the invariance under coordinate dependent transformation we require the existance of a T = 1 vector field B_μ. B_μ is a 2 $\times$ 2 matrix field given by,

$$B_\mu = \frac{1}{\sqrt{2}} \sum_{a=1}^{3} \tau_a B_{\mu a}. \tag{2.39}$$

The nucleon field transforms as,

$$\psi'(x) = U\psi(x), \tag{2.40}$$

whereas the field B_μ is assumed to transform as,

$$B'_\mu (x) = U \left[B_\mu(x) + \frac{1}{ig} \partial_\mu\right] U^{-1}, \tag{2.41}$$

where,

$$U = \exp \left[i\tau/2. \underset{\sim}{\epsilon} (x) \right] , \tag{2.42}$$

and g is the coupling constant. Using (40) and (41) one can

see that the covariant derivative,

$$D_\mu \psi = (\partial_\mu + ig\, B_\mu)\psi \,, \tag{2.43}$$

transforms as,

$$D_\mu \psi' = U\, D_\mu \psi . \tag{2.44}$$

The antisymmetric field tensor analogous to $F_{\mu\nu}$ is,

$$\begin{aligned} G_{\mu\nu} &= D_\mu B_\nu - D_\nu B_\mu \\ &= \partial_\mu B_\nu - \partial_\nu B_\mu - ig\, [\, B_\mu, B_\nu]_- . \end{aligned} \tag{2.45}$$

The kinetic energy term for B_μ field is,

$$-\frac{1}{4}\ \mathrm{Tr}\,(G_{\mu\nu} G_{\mu\nu}) \ . \tag{2.46}$$

The Yang-Mills trick was later generalised to other Lie groups of internal symmetries by Utiyama[8] and Gell-Mann and Glashow[8]. The gauge fields required to maintain invariance have the transformation properties analogous to (41). Since these fields are introduced to cancel the terms $\partial_\mu \epsilon_i$, we have as many gauge fields as there are group parameters; so they belong to the regular representation of the group.

In SU(3) X SU(3) there are sixteen gauge fields belonging to (8, 1) + (1, 8) representation. Out of these sixteen fields, eight are vector Y_μ and other eight are axial vector Z_μ. The 3×3 matrix fields Y_μ and Z_μ are defined as,

$$Y_\mu = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i\, y_{\mu i} \,, \quad Z_\mu = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i\, z_{\mu i} . \tag{2.47}$$

The fields X^+_μ belonging to (8, 1) and X^-_μ belonging to (1, 8)

representation are,

$$X^{\pm}_{\mu} = \frac{1}{\sqrt{2}} (Y_{\mu} \pm Z_{\mu}). \tag{2.48}$$

Using (30), (32) and (41), we obtain for the transformation properties of $X^{\pm}$

$$U_{+}U_{-}X^{\pm}_{\mu}U_{-}^{-1}U_{+}^{-1} = \exp [i\epsilon_{\pm}(x).\lambda/2] [X^{\pm}_{\mu} + \frac{1}{ig} \partial_{\mu}\epsilon_{\pm}]$$
$$\times \exp [-i\, \epsilon_{\pm}(x).\lambda/2]. \tag{2.49}$$

For infinitesimal ϵ, this reads,

$$\delta X^{\pm}_{\mu} = i[\epsilon_{\pm}(x).\lambda/2, X^{\pm}_{\mu}]_{-} - \frac{1}{g} \partial_{\mu}\epsilon_{\pm}(x).\lambda/2. \tag{2.50}$$

The covariant derivative for M field is,

$$D_{\mu}M = \partial_{\mu}M + igX^{-}_{\mu}M - igMX^{+}_{\mu}$$
$$= \partial_{\mu}M + \frac{ig}{\sqrt{2}} [Y_{\mu}, M]_{-} - \frac{ig}{\sqrt{2}} [Z_{\mu}, M]_{+} . \tag{2.51}$$

It transforms as,

$$(D_{\mu}M)' = \exp [i\epsilon_{-}(x).\lambda/2] D_{\mu}M \exp [-i\epsilon_{+}(x).\lambda/2]. \tag{2.52}$$

It was shown by Glashow and Gell-Mann[8] that in general one needs as many coupling constant as there are simple groups. In the present case our group is a direct product of $SU_{+}(3)$ and $SU_{-}(3)$ so we need two coupling constants to write covariant derivatives. However, since these two groups are connected to each other through parity transformation, therefore, only one coupling constant is required. The covariant derivative for M^{+} field is,

$$D_{\mu}M^{+} = \partial_{\mu}M^{+} + \frac{ig}{\sqrt{2}} [Y_{\mu}, M^{+}]_{-} + \frac{ig}{\sqrt{2}} [Z_{\mu}, M^{+}]_{+} . \tag{2.53}$$

It transforms as,

$$(D_\mu M^+)' = \exp\left[\, i\epsilon_+(x).\lambda/2 \,\right] D_\mu M^+ \exp\left[\, -i\epsilon_-(x).\lambda/2 \,\right]. \tag{2.54}$$

Thus,

$$-\frac{1}{2}\,\mathrm{Tr}\,(D_\mu M\, D_\mu M^+), \tag{2.55}$$

is invariant under SU(3) X SU(3) coordinate dependent gauge transformations. The antisymmetric field tensor for fields $X^{\pm}_{\mu\nu}$ are given by,

$$X^{\pm}_{\mu\nu} = \partial_\mu X^{\pm}_\nu - \partial_\nu X^{\pm}_\mu + ig\left[\, X^{\pm}_\mu ,\, X^{\pm}_\nu \,\right], \tag{2.56}$$

which transforms as,

$$(X^{\pm}_{\mu\nu})' = \exp\left[\, i\epsilon_{\pm}(x).\lambda/2 \,\right] X^{\pm}_{\mu\nu} \exp\left[\, -i\epsilon_{\pm}(x).\lambda/2 \,\right]. \tag{2.57}$$

The invariant kinetic energy term for the vector and axial vector fields are,

$$-\frac{1}{4}\quad \mathrm{Tr}\,(X^+_{\mu\nu}X^+_{\mu\nu} + X^-_{\mu\nu}X^-_{\mu\nu}). \tag{2.58}$$

Some other possible invariants are,

$$\mathrm{Tr}\,(X^+_{\mu\nu}M^+\, X^-_{\mu\nu}M), \tag{2.59}$$

and

$$\mathrm{Tr}\,(X^+_{\mu\nu}X^+_{\mu\nu}M^+M + X^-_{\mu\nu}X^-_{\mu\nu}MM^+). \tag{2.60}$$

Other invariant terms involving more derivatives can also be constructed. For example, using (53), (54) and (57), it is easy to see that the interaction term,

$$\begin{aligned} i\,\mathrm{Tr}\,(X^+_{\mu\nu}(D_\mu M^+ D_\nu M - D_\nu M^+ D_\mu M) \\ + X^-_{\mu\nu}(D_\mu M\, D_\nu M^+ - D_\nu M\, D_\mu M^+)), \end{aligned} \tag{2.61}$$

is SU(3) X SU(3) invariant and hermitian.

2-3 Vacuum Breaking of Chiral Symmetries:

An important feature of our model is that we allow the possibility of vacuum being non-invariant under the symmetry transformations. When this is the only mode of symmetry breaking, we say that symmetry is broken spontaneously. Non-invariance of vacuum imposes some constraints, known as Goldstone theorem, on the theory. This theorem states that in a Lorentz covariant theory, the Lagrangian or Hamiltonian is invariant under a continuous group of transformations but vacuum is not, there must exist massless particles, known as Goldstone particles. The existence of such particles was first conjectured by Goldstone[9] and later, this theorem was proved by many authors[9,10]. The Goldstones are spinless if symmetry under considerations is internal one (the symmetry transformations do not involve space time coordinates) such as isospin, SU(3) etc. and carry spin if the symmetry transformations involve space time coordinates such as Lorentz invariance. It can be shown that the quantum numbers of Goldstones are those of nonconserved generators of the original group[20]; corresponding to each nonconserved component of the symmetry, we have one massless particle. Since in nature there are no massless spin zero mesons this type of symmetry breaking is not very useful as such.

Many attempts have been made to bypass the consequences of Goldstone theorem. It was shown by Higgs[11] and Kibble[12] that on introducing vacuum breaking in a gauge theory, where gauge fields are coupled to conserved currents, Goldstones become longitudinal modes of gauge fields and are completely removed from the theory; the corresponding gauge fields acquire mass. In this manner only those gauge fields acquire mass which are coupled to the nonconserved component of the symmetry.

In the present work, we would be breaking SU(3) X SU(3) symmetry spontaneously to SU(2) X U(1), corresponding to isospin and hypercharge symmetry. It means that, if the Lagrangian is completely symmetric, the whole of pseudoscalar octet and the scalar kaon will appear as Goldstones. When the Lagrangian is invariant under coordinate dependent transformations, Goldstones get coupled to corresponding gauge fields and are completely eliminated from the model. Furthermore, ρ(Y=0, I=1) and ϕ(Y=0, I=0) members of vector meson octet remain massless. Since the particles having the quantum numbers of Goldstones are observed in nature, we would certainly like to keep them in the model. In order to overcome these difficulties, a common mass term for the gauge fields i.e.,

$$-\frac{1}{2} m_0^2 \left(Y_\mu Y_\mu + Z_\mu Z_\mu\right), \qquad (2.62)$$

is added to the Lagrangian. This term is invariant only under

SU(3) X SU(3) coordinate independent transformations. Since in presence of vacuum breaking gauge fields and Goldstone fields mix, we define a physical spin one field which is a linear combination of gauge field and derivative of Goldstone field. When Lagrangian is expressed in terms of physical spin one field the m_0^2 term besides giving mass to ρ and ϕ, provides kinetic energy term for Goldstones and thus prevent their complete elimination.

Although Goldstones are not removed from the theory, these are still massless, a situation not in confirmity with experiments. To give mass to these Goldstones one has to add appropriate symmetry breaking term to the Lagrangian. In the presence of this term Goldstone theorem obviously fails. The form of the explicit symmetry breaking term is determined from the requirement that nonconserved currents should satisfy the partial conservation equations. The only explicit breaking term which gives equation like (1.1) is,

$$\mathrm{Tr}\ (B(M + M^{+})), \tag{2.63}$$

which transforms as $(3, 3^{*}) + (3^{*}, 3)$ representation. Thus there are three types of symmetry breaking to the otherwise gauge invariant Lagrangian: a common mass term of gauge fields breaking gauge invariance, then there is a explicit symmetry breaking term giving partial conservation of non-conserved currents and finally symmetry is broken by the vacuum.

CHAPTER III

EFFECTIVE LAGRANGIAN AND MESON MASSES

In this chapter, the invariants constructed in the previous chapter are used to construct the basic Lagrangian for spin one and spin zero mesons. Symmetry breaking is introduced by giving nonzero vacuum expectation values to some scalar fields. After considering field mixings, due to symmetry breaking, and renormalizations, physical fields for various particles are defined. Mass formulae of various particles are obtained from the effective Lagrangian. Finally, all but two parameters entering the model are determined from the experimental spin one and pseudoscalar masses.

3-1 The Effective Lagrangian:

We assign $(3, 3^*) + (3^*, 3)$ representation of $SU(3) \times SU(3)$ to spin zero mesons. The scalar 3×3 matrix field Σ and pseudoscalar matrix field Π are,

$$\Sigma = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i \sigma_i, \quad \Pi = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i \pi_i, \tag{3.1}$$

whereas the vector matrix field Y_μ and axial vector matrix field Z_μ are,

$$Y_\mu = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i y_{\mu i}, \quad Z_\mu = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i z_{\mu i}. \tag{3.2}$$

Let,

$$M = \Sigma + i\Pi \, , \quad \text{and} \quad M^+ = \Sigma - i\Pi \, . \tag{3.3}$$

The relevant invariants of the fields M, M^+, Y_μ, Z_μ have been constructed in the Chapter II. The basic Lagrangian is,

$$L = L_1 + L_2 + L_3 + L_4 + L_5 \, , \tag{3.4}$$

where,

$$L_1 = -\frac{1}{2}\left\{ D_\mu M \, D_\mu M^+ \right\} = -\frac{1}{2}\left\{ (D_\mu \Pi)^2 + (D_\mu \Sigma)^2 \right\}$$

$$L_2 = -\frac{1}{2}\left\{ \mu_o^2 \, W_2 + \alpha W_3 + \beta W_4 + \gamma (W_2)^2 \right\}$$

$$L_3 = -\frac{1}{4}\left\{ F_{\mu\nu} F_{\mu\nu} + G_{\mu\nu} G_{\mu\nu} \right\} - \frac{1}{2} m_o^2 \left\{ Y_\mu Y_\mu + Z_\mu Z_\mu \right\}$$

$$L_4 = \frac{ih}{4} \left\{ X^+_{\mu\nu} (D_\mu M^+ D_\nu M - D_\nu M^+ D_\mu M) \right.$$

$$\left. + X_{\mu\nu} (D_\mu M \, D_\nu M^+ - D_\nu M \, D_\mu M^+) \right\}$$

$$- \frac{h_1}{8} \left\{ X^+_{\mu\nu} X^+_{\mu\nu} M^+ M + X_{\mu\nu} X_{\mu\nu} M M^+ \right\}$$

$$- \frac{h_2}{4} \left\{ X^+_{\mu\nu} M^+ X_{\mu\nu} M \right\}$$

$$= \frac{h}{2\sqrt{2}} \left\{ i\, F_{\mu\nu} \left([\, D_\mu\Sigma,\ D_\nu\Sigma\,]_- + [\, D_\mu\Pi,\ D_\nu\Pi\,]_- \right) + G_{\mu\nu} \left([\, D_\mu\Pi,\ D_\nu\Sigma\,]_+ - [\, D_\nu\Pi,\ D_\mu\Sigma\,]_+ \right) \right\}$$

$$- \frac{h_1}{4} \left\{ (F_{\mu\nu}F_{\mu\nu} + G_{\mu\nu}G_{\mu\nu})(\Sigma^2 + \Pi^2) + i\,[\, F_{\mu\nu},\ G_{\mu\nu}\,]_+ \,[\,\Sigma, \Pi\,]_- \right\}$$

$$- \frac{h_2}{4} \left\{ (F_{\mu\nu} + G_{\mu\nu})(\Sigma - i\Pi)(F_{\mu\nu} - G_{\mu\nu})(\Sigma + i\Pi) \right\}$$

$$L_5 = \left\{ B\Sigma \right\}$$

$$D_\mu\Sigma = \frac{1}{2}(D_\mu M + D_\mu M^+) = \partial_\mu\Sigma + \frac{ig}{\sqrt{2}}[\, Y_\mu,\ \Sigma\,]_- + \frac{g}{\sqrt{2}}[\, Z_\mu,\ \Pi\,]_+$$

$$D_\mu\Pi = \frac{1}{2i}(D\, M - D\, M^+) = \partial_\mu\Pi + \frac{ig}{\sqrt{2}}[\, Y_\mu,\ \Pi\,]_- - \frac{g}{\sqrt{2}}[\, Z_\mu,\ \Sigma\,]_+$$

$$W_2 = \left\{ MM^+ \right\} = \left\{ \Pi^2 + \Sigma^2 \right\}$$

$$W_3 = 3(\det M + \det M^+) = 2\{\Sigma^3\} - 3\{\Sigma\}\{\Sigma^2\} + \{\Sigma\}^3 - 6\{\Sigma\Pi^2\} - 6\{\Sigma\Pi\}\{\Pi\} + 3\{\Sigma\}\{\Pi^2\} - 3\{\Sigma\}\{\Pi\}^2$$

$$W_4 = \left\{ MM^+MM^+ \right\} = \left\{ \Sigma^4 + \Pi^4 + 4\Sigma^2\Pi^2 - 2\Sigma\Pi\Sigma\Pi \right\}$$

$$F_{\mu\nu} = \frac{1}{\sqrt{2}}(X^+_{\mu\nu} + X^-_{\mu\nu}) = \partial_\mu Y_\nu - \partial_\nu Y_\mu + \frac{ig}{\sqrt{2}}[\, Y_\mu,\ Y_\nu\,]_- + \frac{ig}{\sqrt{2}}[\, Z_\mu,\ Z_\nu\,]_-$$

$$G_{\mu\nu} = \frac{1}{\sqrt{2}}(X^+_{\mu\nu} - X^-_{\mu\nu}) = \partial_\mu Z_\nu - \partial_\nu Z_\mu + \frac{ig}{\sqrt{2}}[\, Y_\mu,\ Z_\nu\,]_- - \frac{ig}{\sqrt{2}}[\, Y_\nu,\ Z_\mu\,]_-$$

$$B = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i\, b_i$$

The symbol $\{\ \}$ is a shorthand notation for trace. Apart from the L_4 term our Lagrangian is the SU(3) X SU(3) version of σ model[6], discussed by Levy[14] and Gasiorowicz and Geffen[21], extended further to include spin one particles introduced as gauge fields. The L_4 which involves nonminimal coupling terms of spin one fields is required to describe correctly masses of the gauge fields and decay of axial vector mesons.

In the model we have only octets of vector and axial vector mesons. One can include ninth vector and axial vector mesons as well. However, it requires introducing of many new parameters so that the model looses much of its predictive power. We will see that even without ninth vector and axial vector mesons model give good description of the experimental data. The inclusion of ninth vector and axial vector meson is discussed in detail in sixth chapter.

The presence of L_5 term implies that the field Σ couples to vacuum, and leads to tadpole graphs[22]. Let,

$$\Sigma = f + \Phi \qquad (3.5)$$

such that $\langle 0|\Phi|0\rangle = 0$. The matrix f is defined as,

$$f \equiv \langle 0|\Sigma|0\rangle = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i \langle\sigma_i\rangle_0 . \qquad (3.6)$$

It follows from charge conservation that only $\langle\sigma_0\rangle_0$, $\langle\sigma_8\rangle_0$, $\langle\sigma_3\rangle_0$, $\langle\sigma_{6,7}\rangle_0$ can be nonzero. The $\langle\sigma_0\rangle_0$ breaks

SU(3) X SU(3) to SU(3) and $\langle\sigma_8\rangle_o$ to SU(2) X U(1) corresponding to isospin and hypercharge symmetry. $\langle\sigma_3\rangle_o$ breaks isospin whereas $\langle\sigma_{6,7}\rangle_o$ break hypercharge as well as isospin symmetry. Since in the present work we will be considering only strong interactions, only $\langle\sigma_o\rangle_o$ and $\langle\sigma_8\rangle_o$ are nonzero so that we can write,

$$f = \frac{1}{\sqrt{2}} \left(f_o I + f_8 \lambda_8 \right) \tag{3.7}$$

When (7) is substituted in (4), L_1 gives term like $Y . \partial_\mu \Phi$ and $Z_\mu . \partial_\mu \Pi$. To take account of this mixing we write,

$$\begin{aligned} y^i_\mu &= \hat{y}^i_\mu + \xi^s_{ij} \partial_\mu \phi_j , \\ z^i_\mu &= \hat{z}^i_\mu + \xi^p_{ij} \partial_\mu \pi_j , \end{aligned} \tag{3.8}$$

where $\hat{y}_\mu$ and $\hat{z}_\mu$ are the new vector and axial vector fields respectively. The coefficients $\xi^{p,s}_{ij}$ are to be determined such that terms like $\hat{y}_\mu . \partial_\mu \phi$ and $\hat{z}_\mu . \partial_\mu \pi$ are absent from the Lagrangian. This gives,

$$\begin{aligned} \xi^s_{ij} &= g\, f_{ij8}\, f_8 / (m_o^{\,2} + g^2 f_{8ik} f_{kj8} f_8^2) \\ \text{and} \quad \xi^p_{ij} &= g\, d_{ijk}\, f_k / (m_o^{\,2} + g^2 d_{ilk} d_{jln} f_k f_n). \end{aligned} \tag{3.9}$$

In the pseudoscalar meson kinetic energy, there is mixing between 0 and 8 components which can be removed by introducing new $\hat{\pi}_o$ and $\hat{\pi}_8$ fields through the relation,

$$\begin{aligned} \pi_8 &= \hat{\pi}_8 \cos\psi + \hat{\pi}_o \sin\psi \\ \pi_o &= -\hat{\pi}_8 \sin\psi + \hat{\pi}_o \cos\psi \end{aligned} \tag{3.10}$$

The requirement that there should not be any terms like $\partial_\mu \hat{\pi}_0 \cdot \partial_\mu \hat{\pi}_8$ gives,

$$\tan 2\psi = -2\sqrt{2/3}\; f_8(f_0 - f_8/\sqrt{3})/(f_0^2 - 2f_0 f_8/\sqrt{3} - f_8^2/3). \quad (3.11)$$

Since the substitution (8) modifies the kinetic energy of spin zero mesons, we introduce renormalized scalar fields s_i and pseudoscalar fields p_i through the relations,

$$\begin{aligned} s_i &= Z_{s_i}^{-1/2}\, \phi_i\,, & i &= 0, \ldots, 8 \\ p_i &= Z_{p_i}^{-1/2}\, \pi_i\,, & i &= 1, \ldots, 7 \\ p_i &= Z_{p_i}^{-1/2}\, \hat{\pi}_i\,, & i &= 0, 8 \end{aligned} \qquad (3.12)$$

and determine Z's such that coefficient of kinetic energy term for s and p fields are $-1/2$. This gives,

$$\begin{aligned} Z_{s_i} &= 1 + g^2 f_8^2\, f_{i8j}\, f_{j8i} / m_0^2\,, \qquad i = 0, \ldots, 8 \\ Z_{p_i} &= 1 + g^2[(f_0 + f_8\, d_{ii8})^2 + 2f_8^2 \delta_{i8}/3] / m_0^2 \\ & \qquad\qquad i = 1, \ldots, 8 \\ Z_{p_0} &= 1 \end{aligned} \qquad (3.13)$$

The symmetry breaking also modifies the kinetic energy of spin one mesons. As before the renormalized vector meson field v_μ^i and axial vector meson fields a_μ^i are defined through the relation,

$$\begin{aligned} \hat{y}_\mu^i &= Z_{V_i}^{1/2}\, v_\mu^i\,, \\ \hat{z}_\mu^i &= Z_{A_i}^{1/2}\, a_\mu^i\,. \end{aligned} \qquad (3.14)$$

The renormalization constants Z_{V_i} and Z_{A_i} are given by,

$$
\begin{aligned}
Z_{V_i}^{-1} &= 1 + \frac{h_1}{2} d_{iik}\, d_{klm}\, f_l\, f_m \\
&\quad + \frac{h_2}{2} (d_{ijk} + if_{ijk})(d_{ilk} + if_{ilk})\, f_j\, f_l , \\
Z_{A_i}^{-1} &= 1 + \frac{h_1}{2} d_{iik}\, d_{klm}\, f_l\, f_m \\
&\quad - \frac{h_2}{2} (d_{ijk} + if_{ijk})(d_{ilk} + if_{ilk})\, f_j\, f_l .
\end{aligned}
\tag{3.15}
$$

In the effective Lagrangian, there should be no terms linear in physical scalar field s. This requires that only b_o and b_8 are nonzero and satisfy the two conditions,

$$
f_o \underset{\sim}{\mu}_o^2 + \frac{3\alpha}{\sqrt{2}} (f_o^2 - f_8^2/3) + \beta(f_o^3 + 2f_o f_8^2 - 2f_8^3/3\sqrt{3}) - \sqrt{2/3}\, b_o = 0 , \tag{3.16}
$$

and
$$
\underset{\sim}{\mu}_o^2 - \frac{3\alpha}{\sqrt{2}} (f_o + f_8/\sqrt{3}) + 3\beta(f_o^2 - f_o f_8/\sqrt{3} + f_8^2/3) - b_8/f_8 = 0 , \tag{3.17}
$$

where,

$$
\underset{\sim}{\mu}_o^2 = \mu_o^2 + (3f_o^2 + 2f_8^2) . \tag{3.18}
$$

3-2 Particle Masses:

In our model we have octets of vector and axial vector mesons and nonets of scalar and pseudoscalar mesons. We identify I = 1, Y = 0 member of vector octet with ρ(765), and I = 1/2, Y = $\pm$ 1 member with K^*(890) resonance.[23] For I = 0, Y = 0 member there are two candidates, ω(783) and ϕ(1019). We will see that our model accommodates ϕ as the

eight member of the vector meson multiplet. It is in agreement with the general view that ϕ belongs more to octet[24]. The I = 1, Y = 0 member of axial vector octet is identified with A_1 (1070). In the region 1240 - 1400 MeV several I = 1/2, Y = $\pm$ 1 peaks have been reported. Although spin parity assignments of these peaks are not conclusive but all the experiments favour 1^+. We assume that one of the resonance in this region is to be identified with K_A, I = 1/2, Y = $\pm$ 1, member of the axial vector octet. As we shall see in next section that our model favours K_A mass around 1310 MeV which could be identified with the corresponding reported peak at 1320 MeV. For I = 0, Y = 0 member of the axial vector meson octet, there are two possible candidates D(1285) and E (1420). Again spin parity of these two resonances are not fixed but 1^+ is favoured over other assignments. Our model favours E as the eight member of the axial vector meson octet.

In the pseudoscalar nonet, we identify, as usual, the I = 1, Y = 0; I = 1/2, Y = $\pm$ 1 and I = 0, Y = 0 members with π (140), K (494), η (549) and X^0 (958) respectively. The situation about the scalar mesons is not at all clear yet. Some experiments suggest existence of an I = 1, Y = 0 scalar meson, δ, at 960 MeV, but is not confirmed by others. Another possible candidate for this member is π_N at 1016 MeV. Our model favours a scalar pion around 960 MeV. The question of

existence of scalar kaon is discussed by several authors. It may lie either below the K-π threshold or above it. If it lies below the K-π threshold its decay mode would be $S_K \rightarrow K + 2\gamma$ which makes its detection difficult. On the other hand, if it lies around 1000 MeV, then the presence of nearly K^* (890) would make its detection difficult. A scalar kaon was reported in the range 1080 - 1260 MeV which is still in doubt. Our model predict a scalar kaon around 1050 MeV. For the other members of scalar nonet i.e. $I = 0$, $Y = 0$, members so far no convincing experimental support exists. In literature, some authors[25] while discussing the $K \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ decays have postulated existence of a scalar meson around 400 MeV with width 100 MeV. So far such a meson has not been confirmed or excluded by experiments.[26] The reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ has been analysed by many authors[23] who find that $I = 0$, s-wave π-π phase shift δ_0^0 passes through $\pi/2$ in the region 650 - 900 MeV. 'Although no method of π-π phase shift analysis used is free from serious objections, the fact that all analysis find the s-wave phase shift δ_0^0 to be near $\pi/2$ in the region 650 to 900 MeV is quite impressive.'[23] The width of this resonance is well over 100 MeV. Another possible candidate for the $I = 0$, $Y = 0$, member of the scalar nonet is η_{0+} at 1060. The predictions of our model for these members are around 1100 MeV and 700 MeV.

The masses of the members of vector multiplet are,

$$
\begin{aligned}
M_\rho^2 &= Z_\rho\, m_o^2 \quad , \\
M_\phi^2 &= Z_\phi\, m_o^2 \quad , \\
M_{K^*}^2 &= Z_{K^*}\,[m_o^2 + 3g^2 f_8^2/4] \ .
\end{aligned}
\tag{3.19}
$$

The renormalization constants as given by (15) are,

$$
\begin{aligned}
Z_\rho^{-1} &= 1 + (h_1+h_2)(f_o+f_8/\sqrt{3})^2/2 \ , \\
Z_{K^*}^{-1} &= 1 + h_1(f_o^2 - f_o f_8/\sqrt{3} + 5f_8^2/6)/2 \\
&\quad + h_2(f_o+f_8/\sqrt{3})(f_o-2f_8/\sqrt{3})/2, \\
Z_\phi^{-1} &= 1 + (h_1+h_2)(f_o^2 - 2f_o f_8/\sqrt{3} + f_8^2)/2 \ .
\end{aligned}
\tag{3.20}
$$

The masses of the members of axial vector multiplet are,

$$
\begin{aligned}
M_{A_1}^2 &= Z_{A_1}\,(m_o^2 + g^2(f_o + f_8/\sqrt{3})^2) \ , \\
M_{K_A}^2 &= Z_{K_A}\,(m_o^2 + g^2(f_o - f_8/2\sqrt{3})^2) \ , \\
M_E^2 &= Z_E\,(m_o^2 + g^2(f_o - f_8/\sqrt{3})^2 + 2g^2 f_8^2/3).
\end{aligned}
\tag{3.21}
$$

where,

$$
\begin{aligned}
Z_{A_1}^{-1} &= 1 + (h_1-h_2)(f_o+f_8/\sqrt{3})^2/2 \ , \\
Z_{K_A}^{-1} &= 1 + h_1(f_o^2 - f_o f_8/\sqrt{3} + 5f_8^2/6)/2 \quad , \\
&\quad - h_2(f_o+f_8/\sqrt{3})(f_o-2f_8/\sqrt{3})/2, \\
Z_E^{-1} &= 1 + (h_1-h_2)(f_o^2 - 2f_o f_8/\sqrt{3} + f_8^2)/2 \ .
\end{aligned}
\tag{3.22}
$$

As expected f_o splits the axial vector multiplet from the vector multiplet, whereas f_8 causes splitting among different isospin multiplets.

The masses of pseudoscalar mesons are,

$$
\begin{aligned}
Z_\pi^{-1} M_\pi^2 &= \mu_o^2 + 3\alpha(f_o-2f_8/\sqrt{3})/\sqrt{2} + \beta(f_o+f_8/\sqrt{3})^2 , \\
Z_K^{-1} M_K^2 &= \mu_o^2 + 3\alpha(f_o+f_8/\sqrt{3})/\sqrt{2} \\
&\quad + \beta(f_o^2-f_of_8/\sqrt{3}+7f_8^2/3) , \\
Z_8^{-1}(M_p^2)_{88} &= M_1 \cos^2\psi + M_2 \sin^2\psi - M_3 \sin 2\psi , \\
Z_o^{-1}(M_p^2)_{oo} &= M_1 \sin^2\psi + M_2 \cos^2\psi + M_3 \sin 2\psi , \\
(Z_8Z_o)^{-1/2}(M_p^2)_{o8} &= (M_1-M_2) \sin 2\psi + 2M_3 \cos 2\psi ,
\end{aligned}
\tag{3.23}
$$

where,

$$
\begin{aligned}
M_1 &= \mu_o^2 + 3\alpha(f_o+2f_8/\sqrt{3})/\sqrt{2} + \beta(f_o^2-2f_of_8/\sqrt{3}+f_8^2) , \\
M_2 &= \mu_o^2 - 6\alpha f_o/\sqrt{2} + \beta(f_o^2+2f_8^2/3) , \\
M_3 &= \sqrt{3}\alpha f_8 + \sqrt{2/3}\, \beta f_8(2f_o-f_8/\sqrt{3}) .
\end{aligned}
$$

The renormalization constants as given by (13) are,

$$
\begin{aligned}
Z_\pi &= 1 + g^2(f_o+f_8/\sqrt{3})^2/m_o^2, \\
Z_K &= 1 + g^2(f_o+f_8/2\sqrt{3})^2/m_o^2 , \\
Z_\eta &= 1 + g^2(f_o^2-2f_of_8/\sqrt{3}+f_8^2)/m_o^2 , \\
Z_{\eta'} &= 1 .
\end{aligned}
\tag{3.24}
$$

Since there is mixing in p_o and p_8 fields, we define physical fields η and η' through the relations,

$$
\begin{aligned}
p_8 &= \eta \cos\theta_p + \eta' \sin\theta_p , \\
p_o &= -\eta \sin\theta_p + \eta' \cos\theta_p .
\end{aligned}
\tag{3.25}
$$

The requirement that there be no mixed terms $\eta\eta'$ gives,

$$\tan 2\theta_p = (M_p^2)_{o8}/((M_p^2)_{oo} - (M_p^2)_{88}) \quad . \tag{3.26}$$

The η and η' masses are,

$$\begin{aligned} 2M_\eta^2 &= (M_p^2)_{oo}+(M_p^2)_{88}-[((M_p^2)_{oo}-(M_p^2)_{88})^2+(M_p^2)_{o8}^2]^{1/2} , \\ 2M_{\eta'}^2 &= (M_p^2)_{oo}+(M_p^2)_{88}+[((M_p^2)_{oo}-(M_p^2)_{88})^2+(M_p^2)_{o8}^2]^{1/2} . \end{aligned} \tag{3.27}$$

The masses of the nine scalar mesons are,

$$\begin{aligned} M_{S_\pi}^2 &= \mu_o^2 - 3\alpha(f_o-2f_8/\sqrt{3})/\sqrt{2} + 3\beta(f_o+f_8/\sqrt{3})^2 , \\ Z_{S_K}^{-1} M_{S_K}^2 &= \mu_o^2 - 3\alpha(f_o+f_8/\sqrt{3})/\sqrt{2} + 3\beta(f_o^2- f_of_8/\sqrt{3}+f_8^2/3) , \\ (M_S^2)_{88} &= \mu_o^2 - 3\alpha(f_o+2f_8/\sqrt{3})/\sqrt{2} \\ &\quad + 3\beta(f_o^2-2f_of_8/\sqrt{3}+f_8^2)+4\gamma f_8^2 , \\ (M_S^2)_{oo} &= \mu_o^2 + 3\sqrt{2}\alpha f_o + \beta(3f_o^2 + 2f_8^2) + 6\gamma f_o^2 , \\ (M_S^2)_{o8} &= -2\sqrt{3}\alpha f_8 + 4\sqrt{6}\beta f_8(f_o-f_8/2\sqrt{3})+4\sqrt{6}\gamma f_of_8 , \end{aligned} \tag{3.28}$$

where,

$$Z_{S_K} = 1 + 3g^2f_8^2/(4m_o^2) \quad . \tag{3.29}$$

To take into account, the mixing of s_o and s_8 fields, we define physical fields S_η and $S_{\eta'}$, the mixing angle θ_S and the physical masses M_{S_η} and $M_{S_{\eta'}}$ through the equations analogous to (25), (26) and (27) respectively.

Expanding meson masses in powers of f_8 and keeping only first order terms in f_8, we see that meson masses

(squared) satisfy the Gell-Mann[1]-Okubo[27] mass formulas,

$$
\begin{aligned}
M^2_{\rho} + 3M^2_{\phi} &= 4M^2_{K^*} , \\
M^2_{\pi} + 3(M^2_{p})_{88} &= 4M^2_{K} , \\
M^2_{S_\pi} + 3(M^2_{S})_{88} &= 4M^2_{S_K} .
\end{aligned} \tag{3.30}
$$

3-3 Determination of Parameters:

In the model there are eleven parameters m_o, g, f_o, f_8, μ_o, α, β, γ, h, h_1 and h_2. All of them, except h, occur in meson masses. The spin one meson masses are functions of m_o, g, f_o, f_8, h_1 and h_2 whereas spin zero masses are functions of g, μ_o, f_8, f_o, α, β and γ. In all spin zero meson masses, except those of S_η and $S_{\eta'}$, the parameters μ_o^2 and γ occur in the combination $\underset{\sim}{\mu}_o^2$ defined by (18). Moreover, g can be absorbed in other parameters such that particle masses, except those of S_η and $S_{\eta'}$, become function of eight parameters namely, m_o, gf_o, ξ ($= f_8/f_o$), h_1/g^2, h_2/g^2, $\underset{\sim}{\mu}_o$, α/g, β/g^2. In addition to these parameters, the S_η and $S_{\eta'}$ masses will involve one more parameter x, defined by,

$$x = - 10\,\gamma/\beta \tag{3.31}$$

In principle, five parameters can be determined from spin one meson masses. It turns out that ξ is a very crucial parameter in our model. Therefore, instead of fixing it from some mass, we vary it in a certain range (it is negative

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Table I: Particle Masses (in MeV) for Some Values of

(S_{η} and $S_{\eta'}$ Masses are for x = 0).

	−0.04	−0.06	−0.08	−0.10	−0.12	Experimental candidates.[a]
M_{K^*}	933	934	935	937	939	890
M_{K_A}	1305	1307	1308	1310	1312	1320
M_{η}	543	549	554	559	564	549
$M_{\eta'}$	961	958	955	952	949	958
M_{S_π}	1670	1315	1096	942	826	1016, 960
M_{S_K}	1741	1395	1187	1043	936	1080 ∼ 1200
M_{S_η}	1488	1139	948	822	728	650 ∼ 900
$M_{S_{\eta'}}$	1783	1445	1245	1111	1012	1060

a. Ref. 23.

if $F_K/F_\pi > 1$), study the variation of various quantities and then select a value of ξ which gives the best overall fit. For other inputs one can take any four spin one meson masses. The eight vector meson can be identified either with ω (783) or ϕ (1019). Similarly for eight axial vector meson, there are two possible candidates D(1286) and E(1420). Out of these four possible combinations ϕ (1019) and E(1420) gives the best fit. For other two inputs, ρ (765) and A_1(1070) are the best inputs. The remaining, three parameters are determined by taking M_π, M_K and $M_\eta^2 + M^2_{\eta'}$ as inputs. Then ξ is varied such that individual η and η' masses are near the respective experimental masses. For some values of ξ predicted particle masses are given in Table I. (S_η and $S_{\eta'}$ masses for x = 0). From the table, we see that the K^*, K_A, η and η' masses do not vary much with ξ, however, scalar masses are sensitive functions of this parameter. For a given ξ a value of x can be obtained from the knowledge of one of the S_η and $S_{\eta'}$ masses. It turns out, however, that π-π scattering lengths depend sensitively on x. Therefore, instead of fixing it from one mass about which experimental situation is not clear, the requirement that s-wave I = 0, π-π scattering length lie between $0 \sim 1$ M_π^{-1}, gives a range for x. With x in this range, the best value of ξ turns out to be -0.098. The various masses for ξ = -0.098 are as follows,

$$M_{K^*} = 936 \text{ MeV} \quad , \quad M_{K_A} = 1310 \text{ MeV} \; ,$$
$$M_{\eta} = 558 \text{ MeV} \quad , \quad M_{\eta'} = 952 \text{ MeV} \; , \qquad (3.32)$$
$$M_{S_\pi} = 956 \text{ MeV} \quad , \quad M_{S_K} = 1055 \text{ MeV} \; .$$

The S_η and $S_{\eta'}$ masses (in MeV) for some values of x are,

x	1.0	1.1	1.2
M_{S_η}	700	685	669
$M_{S_{\eta'}}$	1105	1104	1103
θ_S	15^o	14.5^o	14^o .

(3.33)

The values of other parameters are,

$$\begin{aligned}
m_o &= 468.3 \text{ MeV} \; , \\
gf_o &= 550.8 \text{ MeV} \; , \\
h_1 M_\rho^2/g^2 &= -2.9 \; , \\
h_2 M_\rho^2/g^2 &= -0.114 \; , \\
\mu_o^2 &= -2.486 \times 10^5 \text{ (MeV)}^2 \; , \\
\alpha/g &= -75.1 \text{ MeV} \; , \\
\beta/g^2 &= 1.31 \; , \\
\psi &= 4.3^o \; , \\
\theta_p &= -11.6^o \; .
\end{aligned} \qquad (3.34)$$

The (16), (17) and (34) give,

$$C = b_8/b_o = -1.25 \; . \qquad (3.35)$$

The parameter ξ gives the relative strength of the vacuum breaking of SU(3) and of SU(3) X SU(3) whereas the quantity C is a measure of the relative strengths of the corresponding intrinsic breakings. The latter is close to the SU(2) X SU(2) value ($-\sqrt{2}$) and is same as the value obtained by Gell-Mann, Oakes and Renner[13]. The value of ξ, however, is nowhere close to the SU(2) X SU(2) value ($\xi = -\sqrt{3}$) but is nearer to the SU(3) value ($\xi = 0$) which means that vacuum is approximately symmetric under SU(3) and not under SU(2) X SU(2). The point of view that vacuum is SU(3) symmetric has been emphasized by Dashen and Weinstein.[28] If the vacuum breaking of SU(2) X SU(2) were also negligible, the masses of ρ and A_1 and of π and S_π should be nearly equal. The deviation of ξ from the SU(2) X SU(2) value is therefore a measure of $\rho - A_1$ mass difference whereas the deviation of C from the SU(2) X SU(2) value is a measure of the pion mass. Comparing the experimental value of $\rho - A_1$ mass difference (~ 310 MeV) with the experimental pion mass (~ 140 MeV), we see that the former deviation is expected to be larger on empirical grounds also.

Our value of ξ is smaller than the value obtained by other authors. Glashow, Schnitzer and Weinberg[28] ($\xi = -0.36$) determined it from spin one mass spectrum alone. Levy's[14] determination from spin zero mass spectrum give $\xi = -0.26$. Gasiorowicz and Geffen[16] obtained ξ in the range -0.117 to -0.21 from the consideration of particle masses and meson decay constants.

In an earlier work,[18] we reported on some investigation with this model with $h_1 = h_2 = 0$. One of the prediction that follows from (19) is $M_\rho = M_{V_8}$. In this case V_8 is naturally identified with ω (783). In this model the predicted scalar masses were in the range 400 - 700 MeV. Moreover, the situation of spin one masses was also not satisfactory. With $h_1 = h_2 = 0$, one needs a value of ξ around -0.4 to fit spin one masses whereas scalar masses around 1000 MeV require a value of ξ around -0.1.

CHAPTER IV

MESON COUPLINGS, DECAY WIDTHS AND SCATTERING LENGTHS

In this chapter, the effective Lagrangian and the parameters obtained in the last chapter are used to study the strong processes of mesons. First we consider meson decays involving two particle ($V \to PP$, $A \to VP$, $A \to PS$, $S \to PP$) and three particle final states ($A \to 3P$, $P \to 3P$). Then the elastic scattering of a pseudoscalar meson by a pseudoscalar meson is considered, in particular we calculate the low energy parameters, namely, scattering lengths and effective ranges. All processes are considered in the tree graph approximation only i.e. no diagrams involving loops are considered. The numerical calculations are done on IBM 7044 computer at IIT Kanpur.

4-1 Meson Decays:

The S operator is given by,

$$S = \exp(i\int d^4x\, L_{int}(x)) \quad , \tag{4.1}$$

where L_{int} is the interaction part of the effective Lagrangian. Consider the decay of a particle N of four momentum q into n particles $N_1 \ldots N_n$ of four momenta $q_1 \ldots q_n$ respectively,

$$N(q) \to N_1(q_1) + \ldots + N_n(q_n) \quad . \tag{4.2}$$

The relevant matrix element is,

$$\begin{aligned} < N_1(q_1) \ldots N_n(q_n) \mid S \mid N(q) > &= i(2\pi)^4\delta^4(q-\sum_{i=1}^{n} q_i) \\ &\times (2\pi)^{-3/2}(2E_N)^{-1/2} \prod_{i=1}^{n} ((2\pi)^{-3/2}(2E_i)^{-1/2}) \\ &\times \mathcal{M}(N \to N_1 \ldots N_n) \quad , \end{aligned} \tag{4.3}$$

where E_i denotes the energy of the i-th particle, $\mathcal{M}$ is the invariant matrix element to be calculated from effective Lagrangian in the tree graph approximation. The decay width is,

$$\begin{aligned} \Gamma = \int \ldots \int \frac{d^3q_1 \ldots d^3q_n}{(2\pi)^{3n-4}\; 2E_N} \prod_{i=1}^{n} (2E_i)^{-1} \\ \times \left(\frac{1}{2J+1} \sum_{spin} |\mathcal{M}|^2 \right) \quad , \end{aligned} \tag{4.4}$$

where J is the spin of the decaying particle. In the following we will consider the final states containing two and three particles.

(i) Two Particle Final States:

When the final state consists of two particle, one can perform momentum integrations in (4) explicitly. The decay width, in the rest frame of decaying particle, becomes,

$$\Gamma = \frac{1}{8\pi} \frac{1}{2J+1} \frac{k_{CM}}{M^2} \sum_{spin} |\mathcal{M}|^2 \quad . \tag{4.5}$$

Here M is the mass of the decaying particle and k_{CM} is the centre of mass momentum of the decay products given by,

$$k_{CM}^2 = [M^2-(M_1+M_2)^2][M^2-(M_1-M_2)^2]/4M^2 \quad , \tag{4.6}$$

where M_1 and M_2 are the masses of the particles in the final state.

First we consider the decay of a vector meson into two pseudoscalar mesons,

$$V(q) \to P_1(k_1) + P_2(k_2) \quad . \tag{4.7}$$

The effective coupling for any process of the type (7) is of the form,

$$L(VP_1P_2) = ig_1^{VP_1P_2} v_\mu p_1 \partial_\mu p_2 - ig_2^{VP_1P_2} v_\mu \partial_\mu p_1 p_2 + ig_3^{VP_1P_2} (\partial_\mu v_\nu - \partial_\nu v_\mu)(\partial_\mu p_1 \partial_\nu p_2 - \partial_\nu p_1 \partial_\mu p_2) \quad . \tag{4.8}$$

The full structure of the VPP interaction Lagrangian L(VPP) is given in the appendix. After performing an integration by parts on the last two terms of (8), L(VPP) can be written as,

$$L(VPP) = i(g_1^{VPP} + g_2^{VPP} + 2q^2 g_3^{VPP})\, V_\mu p_1 \partial_\mu p_2 \,. \qquad (4.9)$$

When V_μ is coupled to a conserved current (e.g. ρ, ϕ), $g_1 = g_2$ and for $q^2 = 0$, we obtain the universal SU(3) value for the effective coupling apart from Z's of vector mesons. However, when V_μ, is coupled to a nonconserved current (e.g. K^*) $g_1 \neq g_2$, the effective couplings are different from SU(3) value. This is due to the different field mixings and renormalization of fields corresponding to particles in the final state. The matrix element for the process (7) is,

$$\mathcal{M}(V \to PP) = (g_1^{VPP} + g_2^{VPP} - 2M_V^2 g_3^{VPP})\, \epsilon.k_2 \,, \qquad (4.10)$$

where ϵ is the polarization vector of vector meson. Now,

$$\sum_{spin} |\mathcal{M}(V\to PP)|^2 = (g_1^{VPP}+g_2^{VPP}-2M_V^2 g_3^{VPP})^2\, k_{2\mu}k_{2\nu} \times (\delta_{\mu\nu}+q_\mu q_\nu/M_V^2)$$

$$= (g_1^{VPP}+g_2^{VPP}-2M_V^2 g_3^{VPP})^2\, k_{CM}^2 \,. \qquad (4.11)$$

Consequently decay width given by (5) is,

$$\Gamma(V\to PP) = \frac{1}{24\pi}\frac{k_{CM}^3}{M_V^2}(g_1^{VPP}+g_2^{VPP}-2M_V^2 g_3^{VPP})^2 \,. \qquad (4.12)$$

Next we consider the decay of an axial vector meson into vector and pseudoscalar meson,

$$A(Q) = V(g) + P(k) \,. \qquad (4.13)$$

The matrix element for this process is[30],

$$\mathcal{M}(A\to V+P) = G_S \epsilon^V.\epsilon^A + G_D \epsilon^V.Q\epsilon^A.q \,, \qquad (4.14)$$

where ϵ_μ^V and ϵ_μ^A are the polarization vectors of vector and axial vector mesons respectively. G_S and G_D are the coupling constants corresponding to S-wave and D-wave decay of the axial vector meson. Now,

$$\sum_{\text{spin}} |\mathcal{M}(A\to V+P)|^2 = (G_S\delta_{\mu\nu}+G_D q_\nu Q_\mu)(G_S\delta_{\beta\alpha}+G_D q_\beta Q_\alpha)$$
$$(\delta_{\mu\alpha}+q_\mu q_\alpha/M_V^2)\ (\delta_{\nu\beta}+Q_\nu Q_\beta/M_A^2)$$
$$= G_S^2(3+k_{CM}^2/M_V^2) - 2G_S G_D M_A k_{CM}^2(k_{CM}^2+M_V^2)^{1/2}/M_V^2$$
$$+ G_D^2\, M_A^2 k_{CM}^4/M_V^2 \quad . \tag{4.15}$$

The decay width is obtained from (15) by multiplying,

$$k_{CM}/24\pi M_A^2 \quad . \tag{4.16}$$

The effective coupling for any specific process of the type (13) is of the form,

$$L(AVP) = g_1^{AVP} a_\mu v_\mu p + g_2^{AVP}(\partial_\mu v_\nu - \partial_\nu v_\mu)(a_\mu \partial_\nu p - a_\nu \partial_\mu p)$$
$$+ g_3^{AVP}(\partial_\mu a_\nu - \partial_\nu a_\mu)(v_\mu \partial_\nu p - v_\nu \partial_\mu p)$$
$$+ g_4^{AVP}(\partial_\mu a_\nu - \partial_\nu a_\mu)(\partial_\mu v_\nu - \partial_\nu v_\mu)p \quad . \tag{4.17}$$

Integrating partially, last three terms of (17), we obtain, on the mass shell of particles involved,

$$G_S = g_1^{AVP} + 2M_V^2 g_2^{AVP} + 2M_A^2 g_3^{AVP} + 2(g_2^{AVP}+g_3^{AVP}+g_4^{AVP})Q.q,$$
$$G_D = -2\,(g_2^{AVP} + g_3^{AVP} + g_4^{AVP}) \quad . \tag{4.18}$$

Other two particle decay mode of axial vector meson is,

$$A(Q) \to P(p) + S(k) \quad . \tag{4.19}$$

The relevant coupling is of the form,

$$L(APS) = g_1^{APS} a_\mu \partial_\mu p s - g_2^{APS} a_\mu p \partial_\mu s$$
$$+ g_3^{APS} (\partial_\mu a_\nu - \partial_\nu a_\mu)\ (\partial_\mu p \partial_\nu s - \partial_\nu p \partial_\mu s)\ . \qquad (4.20)$$

A partial integration on last term reduces (20) to,

$$L(APS) = (g_1^{APS} + g_2^{APS} - 2Q^2 g_3^{APS})\ a_\mu \partial_\mu p s\ . \qquad (4.21)$$

The decay width of the process (19) is given by,

$$\Gamma(A \to PS) = \frac{1}{24\pi}\ \frac{k_{CM}^3}{M_A^2}\ (g_1^{APS} + g_2^{APS} + 2M_A^2 g_3^{APS})^2\ . \qquad (4.22)$$

Another two particle mode of interest is decay of a scalar meson into two pseudoscalar mesons,

$$S(Q) \to P(k_a) + P(k_b)\ . \qquad (4.23)$$

The effective coupling for such a process is of the form,

$$L(SPP) = g_1^{sp_1p_2}\ \partial_\mu s \partial_\mu p_1 p_2 + g_2^{sp_1p_2}\ \partial_\mu s p_1 \partial_\mu p_2$$
$$+ g_3^{sp_1p_2}\ s \partial_\mu p_1 \partial_\mu p_2 + g_4^{sp_1p_2}\ s p_1 p_2\ . \qquad (4.24)$$

When two pseudoscalars in the final states are same then $g_1^{sp_1p_2} = g_2^{sp_1p_2}$ otherwise $g_1^{sp_1p_2} \neq g_2^{sp_1p_2}$. This is due to the different field mixings and renormalizations corresponding to the different particles in the final state. The effective coupling constant on mass shell is,

$$g_{eff} = -M_1^2 g_1^{sp_1p_2} - M_2^2 g_2^{sp_1p_2}$$
$$+ \frac{1}{2}(M_s^2 - M_1^2 - M_2^2)(g_3^{sp_1p_2} - g_1^{sp_1p_2} - g_2^{sp_1p_2}) + g_4^{sp_1p_2}\ . \qquad (4.25)$$

Table II: Decay Widths (in MeV) of Two Particle Decay Modes of Vector and Axial Vector Mesons.

δ	0	0.2	0.4	0.6	0.8	1.0	1.2	Experimental value[a]
$\rho\to\pi\pi$	83	94	105	117	129	142	156	125 ± 20
$K^*\to K\pi$[b]	43	47	51	56	60	65	99	50.1 ± 1.8
$\phi\to\bar{K}K$	2.05	2.49	2.98	3.51	4.08	4.49	5.35	3.2
$A_1\to\rho\pi$	222	199	177	157	137	119	102	95 ± 35[c]
$\to\pi S_\eta$[d]	2.2	3.2	4.3	5.6	7.0	8.6	10.4	
$K_A\to K^*\pi$	557	471	392	320	256	199	150	
$\to\rho K$	34.4	23.2	14.4	7.7	3	0.5	0.1	79 ± 13[c]
$\to\pi S_K$	3.4	5.6	8.4	11.9	15.9	20.4	25.6	
$\to K S_\eta$[d]	3.1	5.1	7.7	10.9	14.5	18.7	23.4	
$E\to\bar{K}^*K$[e]	66.2	39.2	19.2	6.2	0.4	1.8	10.0	17 ± 2
$\to\pi S_\pi$	7.8	14.1	22.2	32.1	43.8	57.6	73.2	34 ± 4
$\to\eta S_\eta$[d]	5	8	12	19	25	33	44	

a. Ref. 23.

b. If experimental K^* mass is used in phase space, widths quoted will be multiplied by 0.8.

c. The quoted widths are total widths.

d. Widths are given for x = 1.1.

e. Width given here are calculated with experimental K^* mass in phase space.

The decay width is,

$$\Gamma(s \rightarrow p_1 + p_2) = g_{eff}^2 k_{CM}/(8\pi M_s^2) \quad . \tag{4.26}$$

The allowed two particle decay modes of vector mesons are $\rho \rightarrow \pi\pi$ $K^* \rightarrow K\pi$, and $\phi \rightarrow \bar{K}K$, those of axial vector mesons are $A_1 \rightarrow (\rho\pi, \pi S_\eta)$, $K_A \rightarrow (\rho K, K^*\pi, \pi S_K, KS_\eta)$ and $E \rightarrow (\eta S_\eta, \pi S_\pi)$ and that of scalar mesons are $S_\pi \rightarrow \eta\pi$, $S_K \rightarrow K\pi$, $S_\eta \rightarrow \pi\pi$ and $S_{\eta'} \rightarrow (\pi\pi, \bar{K}K)$. The decay widths of vector mesons are function of two parameters g and δ where the dimensionless parameter δ is defined by,

$$\delta = M_\rho^2 h/g \quad . \tag{4.27}$$

We will see in the next chapter (see Eq. 5.23) that the value of g is,

$$g^2/4\pi = 1.16 \quad . \tag{4.28}$$

With this value of g, we have,

$$\begin{aligned} \Gamma(\rho \rightarrow \pi\pi) &= 83.1 + 51.4\delta + 8.0\delta^2, \\ \Gamma(K^* \rightarrow K\pi) &= 43.1 + 19.3\delta + 2.2\delta^2, \\ \Gamma(\phi \rightarrow \bar{K}K) &= 2.05 + 2.10\delta + 0.53\delta^2 \ . \end{aligned} \tag{4.29}$$

The widths for some values of δ are given in Table II. The experimental ρ- width is obtained for $0.4 \leq \delta$ 1.0. For this range, the $K^* \rightarrow K\pi$ width is larger than the experimental width. The reason for a larger width is that in the calculations, the predicted K^* mass is used. If, however, we use experimental K^* mass then the K^* width is multiplied by a

factor of 0.8 so that the correct experimental width is obtained for $\delta \sim 1.0$. However, we regard the widths quoted above as the true predictions of the model because once the parameter of the model are fixed, values of masses as well as coupling constants automatically follow. It is interesting that without the ninth vector meson, mixing of which with ϕ will alter its width, our model predict correct value for the ϕ width.

The decay widths of those modes of axial vector mesons which involve S_η in the final states also involve the parameter x, The widths which depend only on δ are as follows:

$$\begin{aligned}
\Gamma\ (A_1 \to \rho\pi) &= 222.2 - 119.2\delta + 16.2\delta^2\ , \\
\Gamma\ (K_A \to \rho K) &= 34.4 - 60.5\delta + 26.6\delta^2\ , \\
\Gamma\ (K_A \to K^*\pi) &= 557.2 - 450.6\delta + 92.2\delta^2\ , \\
\Gamma\ (K_A \to \pi S_K) &= 3.4(1 + 1.5\delta)^2\ , \\
\Gamma\ (E \to \pi S_\pi) &= 7.8(1 + 1.7\delta)^2\ .
\end{aligned} \tag{4.30}$$

From phase space considerations $E \to \bar{K}^* K$ decay is not allowed in our model. However, if experimental K^* mass is used in phase space, the width of this decay mode is,

$$\Gamma\ (E \to \bar{K}^* K) = 2(33.1 - 76.5\delta + 44.3\delta^2). \tag{4.31}$$

The widths of other modes of axial vector mesons are,

$$\begin{aligned}
\Gamma(A_1 \to \pi S_\eta) &= (1.+ 0.97\delta)^2 \, (2.21^{+.40}_{-.34}) \, , \\
\Gamma(K_A \to K S_\eta) &= (1 + 1.46\delta)^2 \, (3.07^{+.46}_{-.44}) \, , \\
\Gamma(E \to \eta S_\eta) &= (1 + 1.72\delta)^2 \, (4.5^{+.3}_{-.4}) \, .
\end{aligned} \tag{4.32}$$

The above widths are written such that value with + sign is for $x = 1.2$, without $\pm$ sign for $x = 1.1$ and with - sign for $x = 1.0$.

The decay widths of allowed modes of axial vector mesons for some value of δ are given in the table II. At present, the widths of two body decay modes of A_1 and K_A are not known. The experiments measure decay widths of their three particle modes i.e. $A_1 \to 3\pi$ and $K_A \to K\pi\pi$, therefore, one should compare the widths of three particle modes with experiments. This will be done in the next section. It is interesting that the $\rho\pi$ mode of A_1 is dominant. This is expected because ρ band covers most of the Dalitz plot.[23] If one assumes that most of the $A_1 \to 3\pi$ width is due to $\rho\pi$ mode then our model favours δ between $0.8 \sim 1.8$. The δ around 1 is also favoured by the V→PP width.

The widths of S_K and S_π are,

		Experimental Value
$\Gamma(S_K \to K\pi)$	= 125 MeV,	$\sim$400 MeV
$\Gamma(S_\pi \to \eta\pi)$	= 53 MeV,	5 MeV

whereas S_η and $S_{\eta'}$ widths for some value of x are,

x	0.9	1.0	1.1	1.2	Experimental Value
$\Gamma(S_\eta \to \pi\pi)$	380	425	473	525	Well over 100
$\Gamma(S_{\eta'} \to \pi\pi)$	12	16	22	28	
$\Gamma(S_{\eta'} \to KK)$	1.7	4.0	7.3	11.6	

In the last few years, $\rho\pi\pi$ and $A_1\rho\pi$ couplings have been discussed extensively in the literature. It would not be out of place to give a brief review of these work and compare with our results. The application of current algebra and soft pion techniques to $A_1 \to \rho\pi$ and $\rho \to \pi\pi$ system gives[31],

$$G_S = 2M_\rho^2/F_\pi = 2g_1^{A_1\rho\pi} , \quad G_D = 0 . \qquad (4.33)$$

To obtain the second equality in G_S, we have used (3.19), (5.11), $h_1 = h_2 = 0$ and $M_{A_1}^2 = 2M_\rho^2$. The coupling constants given by (33) gives 800 MeV for the A_1 width as compared to the experimental total width 95 ± 35 MeV. In deriving (33), one assumes that four momenta of the pion to be zero: it amounts to equating A_1 and ρ masses which is a rather large extrapolation. To improve upon the current algebra result two techniques[32,33], within the framework of current algebra and PCAC were developed simultaneously. Other thing common in these two approaches is use of Weinberg sum rules[34]. In one of the approaches[32], often known as semi soft meson technique, it is assumed that some of the form

factors in the matrix elements $< \pi^0 | J_{\mu}^{A(1-i2)} | \rho^+ >$ and $< \pi^0 | J_{\mu}^{V(1+i2)} | A_1^+ >$ obey once substracted and other unsubstracted dispersion relations. The saturation of these dispersion relations by relevant spin one and spin zero mesons and the limit $k_\pi^2 \to 0$, gives $\rho\pi\pi$ and $A_1\rho\pi$ couplings in terms of one unknown. This constant when determined from the ρ width gives reasonable value for $A_1 \to \rho\pi$ width. In the other approach[33], known as the hard meson technique (because all the particles are on mass shell and so not soft), one starts with the consideration of vacuum expectation value of time ordered products of vector and axial vector currents i.e. $< 0 | T(J_\mu^A J_\nu^A J_\lambda^V) | 0 >$. These three point functions are related to two point functions through the Ward identities. As before these two point functions are approximated by the relevant spin zero and one meson poles. Further the assumption that $A_1 - A_1 - \rho$ vertex is almost linear in momenta (smoothness assumption) gives the $A_1\rho\pi$ and $\rho\pi\pi$ vertex again in terms of one unknown.

Our $\rho\pi\pi$ and $A_1\rho\pi$ vertices with $h_1 = h_2 = 0$, are in agreement with the hard meson calculations[33] and those calculated from the effective Lagrangians.[35] Other AVP vertices are different from those calculated from the SU(3) X SU(3) generalization[36] of the Schnitzer-Weinberg approach. The difference arises mainly because they assume conservation of strangeness changing vector current. When it not conserved

it modifies Weinberg first sum rule which plays an important role in these calculations.

For δ around 1.0 the decay of $A_1 \to \rho\pi$ is predominantly S-wave type ($M_\rho^2 G_D/G_S = -0.54$ for $\delta = 1$), a feature shared by effective Lagrangian calculations[35], hard meson calculations,[31] Quark model calculations[37] and calculations based on superconvergence for $A_1\rho \to A_1\rho$ plus current algebra[38]. In contrast, the calculations with the $\rho\pi - \rho\pi$ superconvergent sum rules[30] and with semi soft pion techniques[32] predict it to be predominantly D-wave decay. In our model for $\delta = 3.4$, $A_1 \to \rho\pi$ decay is purely D type. A recent experimental measurement[39] of angular distribution of $\rho^0\pi^-$ gives,

$$|g_T|g_L|^2 = 0.64 \pm 0.25 \quad . \tag{4.34}$$

This means that δ lies between $2.2 \sim 2.8$. This result disagrees with the predominant D-wave and predominant S-wave picture. In the light of this measurement attempts were made to modify the hard meson calculations[40]. In particular one drops the smoothness assumption. Instead of requiring that A-A-ρ vertex is linear in momenta one constructs most general tensor of rank three out of the three momenta available in the problem. This vertex on proper manipulation gives ρ-π-π and $A_1 - \rho - \pi$ vertex in terms of two parameters. In the language of effective Lagrangians this

generality can be realized by constructing invariants such that one obtains couplings involving trilinear and higher terms in momenta. This means that we must have combinations where two $G_{\nu\mu}$ and one $F_{\mu\nu}$ occur together in at least one term in the Lagrangian.

(ii) <u>Three Particle Final State</u>:

Let us now consider decays of the following type,

$$N(q) \rightarrow N_1(q_1) + N_2(q_2) + N_3(q_3) \quad . \tag{4.35}$$

The decay width for the processes of type (35) can be calculated from (4). The width is as follows,

$$\Gamma(N \rightarrow N_1 N_2 N_3) = \frac{1}{(2\pi)^5} \frac{1}{2J+1} \frac{1}{2M_N} \int \frac{d^3\vec{q}_1}{2E_1} \int \frac{d^3\vec{q}_2}{2E_2} \int \frac{d^3\vec{q}_3}{2E_3}$$
$$\times \delta^4 (q-q_1-q_2-q_3) \sum_{\text{spin}} |\mathcal{M}|^2$$
$$= \frac{1}{2J+1} \frac{1}{(2\pi)^3} \frac{1}{8M_N} \int dE_1 \int dE_2 \sum_{\text{spin}} |\mathcal{M}|^2 . \tag{4.36}$$

The limits of these integrations are[41],

$$\begin{aligned} E_1|_{max} &= (M^2+M_1^2-(M_2+M_3)^2)/2M_N \quad , \\ E_1|_{min} &= M_1 \quad , \\ E_2|_{\substack{Max\\Min}} &= \frac{1}{U} \left[(M-E_1)U \pm \left\{(E_1^2-M_1^2)(U^2-2U(M_2^2+M_3^2) \right.\right. \\ &\qquad \left.\left. + (M_2^2-M_3^2)^2)\right\}^{1/2}\right] \quad , \end{aligned} \tag{4.37}$$

where,

$$U = M_N^2 - 2M_N E_1 + M_1^2 .$$

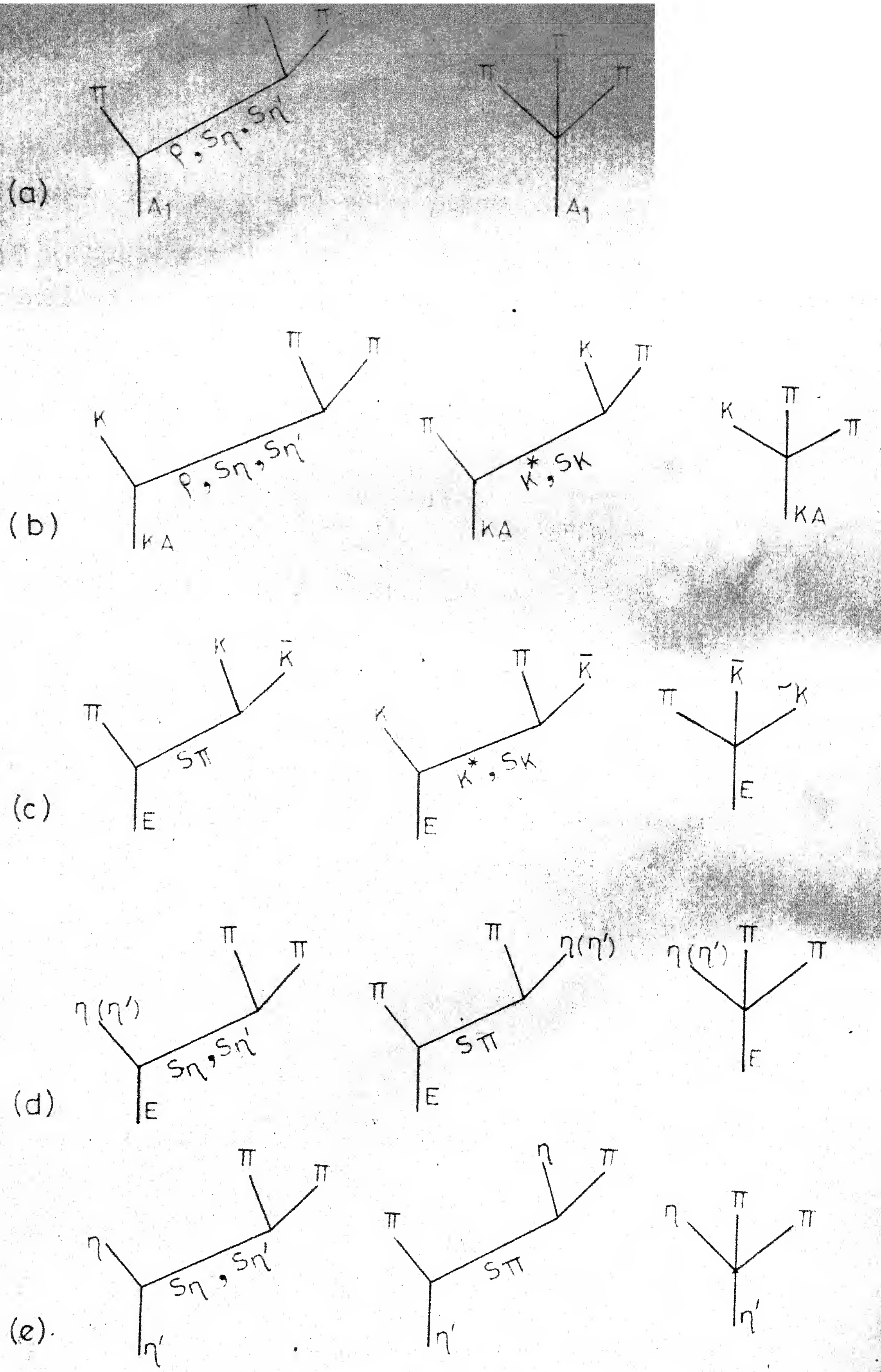

Fig. 1 Tree graphs which contribute to three particle decay modes of mesons. (a) $A_1 \longrightarrow 3\pi$ (b) $K_A \longrightarrow K\pi\pi$ (c) $E \longrightarrow \bar{K}\,K\pi$ (d) $E \longrightarrow \eta(\eta')\pi\pi$ (e) $\eta' \longrightarrow \eta\pi\pi$

We shall consider the following three particle modes, (i) $A_1 \to 3\pi$, (ii) $K_A \to K\pi\pi$, (iii) $E \to (\bar{K}K\pi, \eta\pi\pi, \eta'\pi\pi)$ and (iv) $\eta' \to \eta\pi\pi$. The tree diagrams which contribute to the individual processes are given in Fig. 1.

First consider the decay of A_1 meson into three pions, that is,

$$A_1(q) \to \pi(q_a) + \pi(q_b) + \pi(q_c) \ . \tag{4.38}$$

From isospin invariance, the matrix element $\mathcal{M}$ for this process can be written as,

$$\mathcal{M} = M_1 \, \delta_{Aa} \, \delta_{bc} + M_2 \, \delta_{Ab} \, \delta_{ac} + M_3 \delta_{Ac} \, \delta_{ab}, \tag{4.39}$$

where, A, a, b, c are the isospin indices for A_1 and pions. Further, from the symmetry of (39) we have the following relations among $M_{1,2,3}$,

$$\begin{aligned} M_2(q_a, q_b, q_c) &= M_1(q_b, q_a, q_c) \\ M_3(q_a, q_b, q_c) &= M_1(q_c, q_b, q_a) \ . \end{aligned} \tag{4.40}$$

Now M_1 can be further decomposed as,

$$\begin{aligned} M_1(q_a, q_b, q_c) = {} & M_\rho^{A_1}(q_a, q_b, q_c) + M_s^{A_1}(q_a, q_b, q_c) \\ & + M_c^{A_1}(q_a, q_b, q_c) \ . \end{aligned} \tag{4.41}$$

In writing (41), also throughout this section, the following notations are used. The superscript on M denotes the decaying particle and subscript stand for the intermediate particle connecting two vertices, for example $M_\rho^{A_1}$, $M_s^{A_1}$ are

the contribution from ρ intermediate state and $(S_\eta, S_{\eta'})$ intermediate state respectively. If the subscript is c, it denotes contribution from the contact term. Using the effective Lagrangian for various vertices we obtain,

$$M_\rho^{A_1}(q_a, q_b, q_c) = \frac{i}{(q_a+q_c)^2+M_\rho^2} (g_1^{\rho\pi\pi}+(q_a+q_c)^2 g_3^{\rho\pi\pi}) \cdot$$
$$x\ (\epsilon.q_a(g_1^{A_1\rho\pi} - 4q_b.q_c g_2^{A_1\rho\pi} + 2q.q_b g_3^{A_1\rho\pi} + 4q.q_c g_4^{A_1\rho\pi})$$
$$+\ 2\epsilon.q_b q.(q_c-q_a) g_3^{A_1\rho\pi} + \epsilon.q_c(-g_1^{A_1\rho\pi} + q_a.q_b g_2^{A_1\rho\pi}$$
$$-\ 2q.q_b g_3^{A_1\rho\pi} - 4q.q_a g_4^{A_1\rho\pi}))\ ,$$
$$M_s^{A_1}(q_a, q_b, q_c) = \frac{-i\epsilon.q_a}{(q_c+q_b)^2+M^2_{S_\eta}} ((q_b.q_c-M_\pi^2) g_1^{S_\eta\pi\pi}$$
$$-q_b.q_c g_3^{S_\eta\pi\pi} + g_4^{S_\eta\pi\pi})(g_1^{A_1 S_\eta\pi} + g_2^{A_1 S_\eta\pi} + 2M^2_{A_1} g_3^{A_1 S_\eta\pi}) + S_\eta \leftrightarrow S_{\eta'}\ ,$$
$$M_c^{A_1}(q_a, q_b, q_c) = -i\epsilon.(q_b+q_c) g_1^{A_1\pi\pi\pi}$$
$$+\ i(\epsilon.q_c q_a.q_b + \epsilon.q_b q_a.q_c - 2\epsilon.q_a q_b.q_c) g_3^{A_1\pi\pi\pi}$$
$$-\ iM^2_{A_1}\epsilon.q_a g_3^{A_1\pi\pi\pi}$$
$$-\ i(\epsilon.q_b q.q_a + \epsilon.q_c q.q_a - 2\epsilon.q_a q.(q_c+q_b)) g_4^{A_1\pi\pi\pi}\ . \quad (4.42)$$

The matrix element for the decay $A_1^o \to \pi^o\pi^+\pi^-$ and $A_1^o \to 3\pi^o$ are,

$$\mathcal{M}^{oo+-} = M_1\ ,$$
$$\mathcal{M}^{oooo} = (M_1 + M_2 + M_3)/\sqrt{3!}\ , \quad (4.43)$$

where the factor $\sqrt{3!}$ appears because of identity of three π^o's. We see from (39-43) that to $A_1^o \to 3\pi^o$ only scalar meson intermediate state contribute whereas to $A_1^o \to \pi^+\pi^-\pi^o$ mode all the diagrams contribute.

Next consider the decay,

$$K_A(q) \to K(k) + \pi(p_a) + \pi(p_b) \quad . \tag{4.44}$$

From isospin invariance the matrix elements can be written as,

$$\mathcal{M} = \bar{\chi}_K \left(\delta_{ab} M^+ + \frac{1}{2}\left[\tau_a, \tau_b\right]_- M^-\right) \chi_{K_A}, \tag{4.45}$$

where χ_K and χ_{K_A} are the two component isospin wave functions for K and K_A respectively. As before M^+ and M^- are written as,

$$\begin{aligned} M^+ &= M^+_{K^*} + M^+_{S_K} + M_S^{K_A} + M_c^+ \quad , \\ M^- &= M^-_{K^*} + M^-_{S_K} + M_\rho^{K_A} + M_c^- \quad . \end{aligned} \tag{4.46}$$

In the K_A^+ decay, there can be three different charge states i.e. $K^+\pi^+\pi^-$, $K^+\pi^o\pi^o$ and $K^o\pi^+\pi^o$. The matrix elements for these processes are,

$$\begin{aligned} \mathcal{M}(K_A^+ &\to K^+\pi^+\pi^-) = M^+ + M^- \\ \mathcal{M}(K_A^+ &\to K^+\pi^o\pi^o) = M^+/\sqrt{2} \\ \mathcal{M}(K_A^+ &\to K^o\pi^+\pi^o) = M^- \ , \end{aligned} \tag{4.47}$$

where $\sqrt{2}$ appears due to two identical π^o's. The total width of K_A is the sum of widths of these three modes. Using various

couplings from appendix we obtain,

$$M^{\pm}_{K^*} = M^{K_A}_{K^*}(k,\ p_a,\ p_b) \pm p_a \leftrightarrow p_b$$

$$M^{\pm}_{S_K} = M^{K_A}_{S_K}(k,\ p_a,\ p_b) \pm p_a \leftrightarrow p_b$$

$$M^{K_A}_{S} = \frac{i\epsilon.(p_a+p_b)}{(p_a+p_b)^2+M^2_{S_\eta}}\,(g_1^{K_A S_\eta \pi}+g_2^{K_A S_\eta \pi}+2M^2_{K_A}g_3^{K_A S_\eta \pi})$$

$$x\ ((p_a.p_b-M^2_\pi)g_1^{S_\eta \pi\pi}-p_a.p_b g_3^{S_\eta \pi\pi}+g_4^{S_\eta \pi\pi})+S_\eta \leftrightarrow S_{\eta'}$$

$$M^{K_A}_{\rho}(k,\ p_a,\ p_b) = \frac{-\epsilon.p_a}{(p_a+p_b)^2+M^2_\rho}\,(g_1^{\rho\pi\pi}+(p_a+p_b)^2 g_3^{\rho\pi\pi})$$

$$(g_1^{K_A\rho K}+4k.p_6 g_2^{K_A\rho K}-2q.(k-p_a-p_b)g_3^{K_A\rho K}+4p_b.q g_4^{K_A\rho K})$$

$$-\ p_a \leftrightarrow p_b\quad ,$$

$$M^{+}_{c} = i\epsilon.k(g_1^{K_A K\pi\pi}/2-g_2^{K_A K\pi\pi}-M^2_{K_A}g_6^{K_A K\pi\pi})$$

$$+\ i(\epsilon.p_a k.p_b + \epsilon.p_b k.p_a - 2\epsilon.kp_a.p_b)g_3^{K_A K\pi\pi}$$

$$M^{-}_{c} = 2i(\epsilon.p_a k.p_b - \epsilon.p_b k.p_a)g_4^{K_A K\pi\pi}$$

$$-\ 2i(\epsilon.p_a(q^2-2q.p_b)+\epsilon.p_b(2q.p_a-q^2))g_8^{K_A K\pi\pi}$$

$$+2i(q.p_a\epsilon.p_b-\epsilon.p_a q.p_b)g_9^{K_A K\pi\pi}\ -i\epsilon.(p_a-p_b)g_{10}^{K_A K\pi\pi}$$

where, (4.48)

$$M_{K^*}^{K_A}(k,\ p_a,\ p_b) = \frac{i/2}{(k+p_b)^2+M_{K^*}^2}(\epsilon.p_a(2g_6(p_a.p_b g_2^{K_AK^*\pi}$$

$$+ q.p_b g_3^{K_AK^*\pi} + q.p_b g_4^{K_AK^*\pi}) + g_7(-g_1^{K_AK^*\pi} - 2p_a.p_b g_2^{K_AK^*\pi}$$

$$+2q.(p_a+k)g_3^{K_AK^*\pi} - 2q.p_b g_4^{K_AK^*\pi}) + (g_1^{K_AK^*\pi} + 2M_{K_A}^2 g_3^{K_AK^*\pi})$$

$$x(g_6M_\pi^2+g_7M_K^2-k.p_b(g_6+g_7)/M_{K^*}^2)) + \epsilon.p_b(g_6-g_7)$$

$$x(g_1^{K_AK^*\pi} + 2p_a.(p_b+k)g_2^{K_AK^*\pi} - 2q.p_a g_3^{K_AK^*\pi} + 2q.(p_b+k)g_4^{K_AK^*\pi})),$$

$$M_{S_K}^{K_A}(k,\ p_a,\ p_b) = \frac{-i\epsilon.p_a/2}{(k+p_b)^2+M_{S_K}^2}(g_1^{K_AS_K\pi} + g_2^{K_AS_K\pi} + 2M_{K_A}^2 g_3^{K_AS_K\pi})$$

$$x((k.p_b-M_K^2)g_1^{S_KK\pi} + (k.p_b-M_\pi^2)g_2^{S_KK\pi} - k.p_b g_3^{S_KK\pi} + g_4^{S_KK\pi}),$$

$$g_6 = g_1^{K^*K\pi} + k.(k+p_b)g_3^{K^*K\pi},$$

$$g_7 = g_2^{K^*K\pi} - p_b.(k+p_b)g_3^{K^*K\pi}.$$

In the decay $E \rightarrow \bar{K}K\pi$ i.e. ,

$$E(q) \rightarrow \bar{K}(k_a) + K(k_b) + \pi_c(p)\ , \tag{4.49}$$

the matrix elements are written as,

$$\mathcal{M}(E \rightarrow \vec{\bar{K}}K\pi) = M\bar{\chi}_K \tau_c \chi_{\bar{K}}\ , \tag{4.50}$$

where χ's are two component isospinors. In the final state of E decay, there can be four different final states namely, $E \rightarrow (\bar{K}K^0\pi^0,\ K^+K^-\pi^0,\ K^+\bar{K}^0\pi^-,\ K^0K^-\pi^+)$. From (50) we obtain,

$$\mathcal{M}(E \rightarrow K^+K^0\pi^-) = \mathcal{M}(E \rightarrow K^0K^-\pi^+) = \sqrt{2}\mathcal{M}(E \rightarrow \bar{K}^0K^0\pi^0)$$
$$= \sqrt{2}\mathcal{M}(E \rightarrow K^+K^-\pi^0)\ . \tag{4.55}$$

Therefore, width of E is six times the width of $E \to K^+K^-\pi^o$.

We again write,

$$M = M^E_{K^*} + M^E_{S_K} + M^E_{S_\pi} + M^E_c . \qquad (4.52)$$

The various contributions are,

$$M^E_{K^*} = M_{K^*}(p, k_a, k_b) + k_a \leftrightarrow k_b ,$$

$$M^E_{S_K} = M_{S_K}(p, k_a, k_b) + k_a \leftrightarrow k_b ,$$

$$M^E_{S_\pi} = \frac{-i\epsilon.p}{(q-p)^2+M^2_{S_\pi}} \left(g_1^{ES_\pi\pi} + g_2^{ES_\pi\pi} + 2M_E^2 g_3^{ES_\pi\pi}\right) ,$$

$$x\left(M_K^2 g_1^{S_\pi KK} + k_a.k_b\left(g_3^{S_\pi KK} - g_1^{S_\pi KK}\right) - g_4^{S_\pi KK}\right) ,$$

$$M^E_c = -i\epsilon.p\left(g_1^{EKK\pi} - g_2^{EKK\pi} + 2M_E^2 g_4^{EKK\pi}\right)$$

$$-i(\epsilon.pk_a.k_b - \epsilon.k_a p.k_b - \epsilon.k_b p.k_a) g_3^{EKK\pi} , \qquad (4.53)$$

where $M_{K^*}(p,k_a,k_b)$ and $M_{S_K}(p, k_a, k_b)$ are obtained from $M^{K_A}_{K^*}(k, p_a, p_b)$ and $M^{K_A}_{S_K}(k, p_a, p_b)$, defined by (47), respectively by changing $k \leftrightarrow p$, $p_a \leftrightarrow k_a$, $p_b \leftrightarrow k_b$, $M^2_{K_A} \leftrightarrow M^2_E$ and using appropriate coupling constants for the $E \to \bar{K}^*K$ vertex.

The other three particle modes of E are,

$$E(q) \to \eta(k) + \pi(p_a) + \pi(p_b)$$
$$\to \eta'(k) + \pi(p_a) + \pi(p_b) , \qquad (4.54)$$

The matrix elements for $E \to \eta\pi\pi$ decay are,

$$(E \to \eta\pi\pi) = \frac{+\,i\epsilon.k}{(p_a+p_b)^2+M^2_{S_\eta}} (M^2_\pi g_1^{S_\eta\pi\pi} + p_a.p_b(g_3^{S_\eta\pi\pi} - g_1^{S_\eta\pi\pi})$$

$$+g_4^{S_\eta\pi\pi})(g_1^{E\eta S_\eta} + g_2^{E\eta S_\eta} + 2M_E^2 g_3^{E\eta S_\eta}) + S_\eta \leftrightarrow S_{\eta'}$$

$$+ \frac{i\epsilon.p_a/2}{(k+p_b)^2+M^2_{S_\pi}} ((M_\pi^2+k.p_b)g_1^{S_\pi\eta\pi}$$

$$+(M_p^2 + k.p_b)g_2^{S_\pi\eta\pi} - k.p_b g_3^{S_\pi\eta\pi} + g_4^{S_\pi\eta\pi})$$

$$x(g_1^{E\pi S_\pi} + g_2^{E\pi S_\pi} + 2M_E^2 g_3^{E\pi S_\pi}) + p_a \leftrightarrow p_b$$

$$+i\epsilon.k(2g_1^{E\eta\pi\pi} - g_2^{E\eta\pi\pi} - 2M_E^2 g_3^{E\eta\pi\pi}) \,. \qquad (4.55)$$

The matrix elements for $E \to \eta'\pi\pi$ are obtained from (55) by changing η coupling constants from corresponding η' coupling constants.

The matrix elements for the process,

$$\eta'(q) \to \eta(k) + \pi(p_a) + \pi(p_b) \qquad (4.56)$$

are,

$$\mathcal{M}\,(\eta' \to \eta\pi\pi) = \frac{1}{(p_a+p_b)^2+M^2_{S_\eta}} (M^2_\pi g_1^{S_\eta\pi\pi} + p_a.p_b(g_3^{S_\eta\pi\pi} - g_1^{S_\eta\pi\pi})$$

$$-g_4^{S_\eta\pi\pi})((M^2+k.q)g_1^{S_\eta\eta'\eta} + (M^2{}_{,}+k.q)g_2^{S_\eta\eta'\eta} - k.q g_3^{S_\eta\eta'\eta}$$

$$-g_4^{S_\eta\eta'\eta}) + \frac{1/2}{(k+p_b)^2+M^2_{S_\pi}} (M^2_{\eta'}+q.p_a)g_1^{\eta' S_\pi\pi}$$

$$+(M_\pi^2+q.p_a)g_2^{\eta' S_\pi\pi} - q.p_a g_3^{\eta' S_\pi\pi} - g_4^{\eta' S_\pi\pi})$$

Table III: Decay Widths (in MeV) of Three Particle Decay Mode of Axial Vector Mesons for Some Values of δ and $x = 1.1$.

δ	0	0.2	0.4	0.6	0.8	1.0	1.2	Experimental value[a]
$A_1 \rightarrow 3\pi$	188	177	163	148	133	119	104	95 ± 35
$K_A \rightarrow K\pi\pi$	140	133	126	119	112	106	101	79 ± 13
$E \rightarrow \bar{K}K\pi$	30	53	82	118	161	211	267	69 ± 8
$\rightarrow \eta\pi\pi$	23	26	29	32	33	39	43	$<$ 41
$\rightarrow \eta'\pi\pi$	0.5	0.6	0.7	0.8	0.9	1.0	1.1	

a: Ref. 23.

$$\times(M_\eta^2-k.p_a)g_1^{\eta S_\pi \pi}+(M_\pi^2-k.p_a)g_q^{\eta S_\pi \pi}+k.p_a g_3^{\eta S_\pi \pi}-g_4^{\eta S_\pi \pi})+p_a \leftrightarrow p_b$$
$$+(2k.qg_1^{\eta' \eta \pi\pi}+q.(p_a+p_b)g_2^{\eta' \eta \pi\pi}-k.(p_a+p_b)g_3^{\eta' \eta \pi\pi}$$
$$-2p_a.p_b g_4^{\eta' \eta \pi\pi}+2g_5^{\eta' \eta \pi\pi}) \qquad (4.57)$$

Now while calculating the three particle decay widths numerically it so happens that some of the propagators, like ρ, K^*, S_η $S_{\eta'}$ etc, become singular at some point inside the range of integration. In order to avoid singularities we make finite width approximation, that is in the propagator one makes the substitution $M \to M - i\Gamma/2$ where M is the mass of the intermediate state and Γ its width. The $A_1 \to 3\pi$, $K_A \to K\pi\pi$ and $E \to (\eta', \eta)\pi\pi$ widths are functions of both x and δ, $E \to \bar{K}K\pi$ is function of δ and $\eta' \to \eta\pi\pi$ is function of x only. The widths for some values of δ and x = 1.1 are given in Table III. The variation of $A_1 \to 3\pi$ and $K_A \to K\pi\pi$ widths with x is negligible. The $E \to \eta\pi\pi$ width is changed by 2 MeV if x is changed from 1.2 to 1.0. The $\Gamma(\eta' \to \eta\pi\pi)$ for some values of x is,

x	$\Gamma(\eta' \to \eta\pi\pi)$	Experimental
1.0	2.40	
1.1	1.68	< 2.4
1.2	1.09	

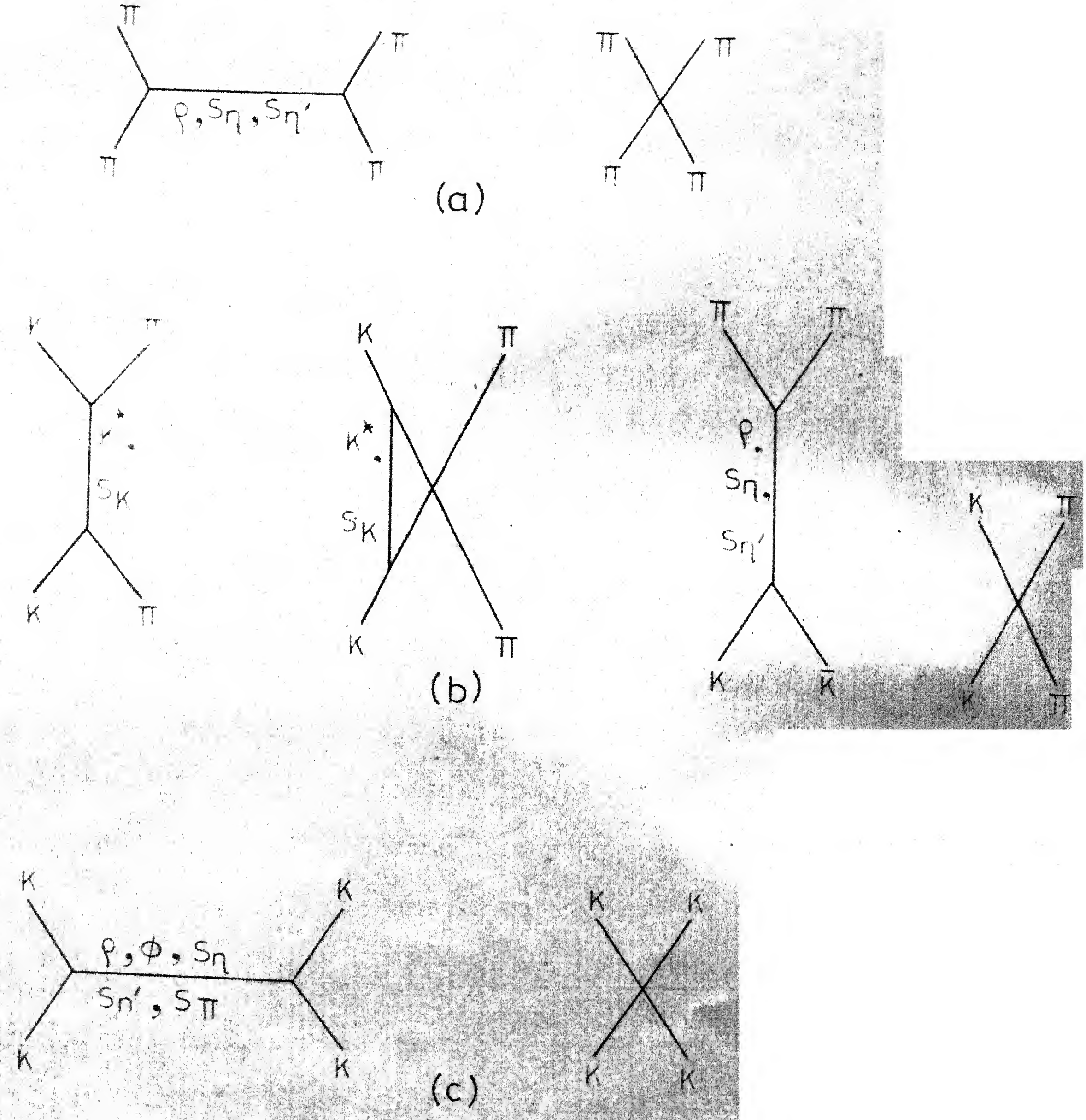

Fig. 2 Diagrams (a), (b), (c) contribute to π − π, K − π and K − K elastic scattering respectively

The experimental $A_1 \to 3\pi$ width favours a value of δ between 0.8 and 1.8. In $A_1 \to 3\pi$ decay ρ intermediate state graph alone accounts 99 percent of the width, in contrast to other works[42] who finds that 3π direct term contribution is about 40 MeV to $A_1 \to 3\pi$ width. It also differ from the calculation[43] who find significant scalar meson contribution. In $K_A \to K\pi\pi$ decay dominating contribution comes from the K^* state. Since $K_A \to K^*\pi$ and $K^* \to K\pi$ widths are large one would expect large $K_A \to K\pi\pi$ width. The small width is indicative of the fact that there is significant interference between various contributions. In $E \to \bar{K}K\pi$ decay dominating contribution comes from the graph with S_π intermediate state. In this mode, the width depends strongly on δ. The reason of this is that K^* intermediate contribution vary slowly with δ whereas S_π state contribution depend strongly on δ.

4-2 Pseudoscalar-Pseudoscalar Scattering:

Now we shall consider the elastic scattering of a pseudoscalar meson by a pseudoscalar meson; in particular $\pi - \pi$, $K - \pi$ and $K - K$ elastic scattering. The diagrams which contribute to these processes are given in Fig. 2. For the process,

$$P_a(q_a) + P_b(q_b) \to P_c(q_c) + P_d(q_d) \ , \tag{4.58}$$

where quantities in the parenthesis denote the four momenta, the scattering amplitude $T(s, t, u)$ is defined as,

$$\langle P_c P_d | S | P_a P_b \rangle = \delta_{fi} + \frac{i\,\delta^4(q_f - q_i)}{(2\pi)^2 \sqrt{(16 E_a E_b E_c E_d)}} T_{ab,cd}(s,t,u), \tag{4.59}$$

where,

$$s = -(q_a + q_b)^2 ,$$
$$u = -(q_a - q_d)^2 ,$$
$$t = -(q_a - q_c)^2 . \tag{4.60}$$

These satisfy the relation,

$$s + t + u = M_a^2 + M_b^2 + M_c^2 + M_d^2 . \tag{4.61}$$

In the centre of mass system (60) becomes,

$$s = (E_a + E_b)^2 = W^2 ,$$
$$u = (E_a - E_b)^2 - 2q_{CM}^2(1 + \cos\theta) ,$$
$$t = -q_{CM}^2(1 - \cos\theta) , \tag{4.62}$$

where q_{CM} and θ are the centre of mass momentum and scattering angle respectively and $E_a^2 = M_a^2 + q_{CM}^2$ etc. The conventional scattering amplitude $F(W, \cos\theta)$ related to the differential cross section through the relation,

$$d\sigma/d\Omega = |F(W, \cos\theta)|^2 , \tag{4.63}$$

is related to $T(s, t, u)$ through the relation,

$$F(W, \cos\theta) = \frac{1}{(2)\,8\pi} \frac{1}{W} T(s, t, u) , \tag{4.64}$$

where the factor of 2 in bracket appears for $\pi - \pi$ and $K - K$ elastic scattering because of identical particles. The sign of right hand side of (64) is determined such that imaginary

part of T satisfy the optical theorem. The partial wave expansion of $F(W, \cos\theta)$ is,

$$F(W, \cos\theta) = \sum_{l=0}^{\infty} (2l+1)\, f_l(W)\, P_l(\cos\theta) \quad . \qquad (4.65)$$

In the elastic region, the partial wave amplitude $f_l(W)$ are related to the phase shifts, δ_l, through the relation,

$$f_l(W) = e^{i\delta_l} \sin\delta_l/|\vec{q}| \quad . \qquad (4.66)$$

The scattering length a_l, which is the value of partial wave amplitude $f_l(W)$ at zero energy, and effective range r_l are defined by the threshold expansion,

$$|\vec{q}|^{2l} \,\mathrm{Re}\, (f_l(W))^{-1} = a_l^{-1} + r_l\, |\vec{q}|^2/2 \quad . \qquad (4.67)$$

(i) <u>$\pi - \pi$ Scattering</u>:

The amplitude $T(s, t, u)$, for the process,

$$\pi_a(p_a) + \pi_b(p_b) \rightarrow \pi_c(p_c) + \pi_d(p_d) \quad , \qquad (4.68)$$

from isospin consideration can be written as,

$$T(s, t, u) = A(s, t, u)\,\delta_{ab}\,\delta_{cd} + B(s, t, u)\,\delta_{ac}\,\delta_{bd} + C(s, t, u)\,\delta_{ad}\,\delta_{bc} \,, \qquad (4.69)$$

where a, b, c, d are the isospin indices of the pions. It is clear from the above decomposition that the amplitudes A, B, C satisfy the crossing relation,

$$B(s, t, u) = A(t, s, u) \,,$$
$$C(s, t, u) = A(u, t, s) \,. \qquad (4.70)$$

The s-channel $I = 0, 1, 2$ isospin amplitudes $T^{0,1,2}$ are related to A,B,C through the relation,

$$\begin{aligned} T^0 &= 3A + B + C \ , \\ T^1 &= B - C \ , \\ T^2 &= B + C \ . \end{aligned} \qquad (4.71)$$

In the tree graph approximation, ρ, S_η, $S_{\eta'}$ exchanges and a contact term contribute to $\pi - \pi$ scattering. Using the effective couplings from the appendix , we obtain (on mass shell),

$$\begin{aligned} A(s, t, u) &= \left\{(g_1^{\rho\pi\pi} + ug_3^{\rho\pi\pi})^2(s-t)/(M_\rho^2-u) + u\leftrightarrow t\right\} \\ &+ (\left\{(s\lambda_A+2\lambda_B)^2/(M^2_{S_\eta}-s)+S_\eta \leftrightarrow S_{\eta'}\right\} + 8g_1^{\pi\pi}) \\ &+ 2g_2^{\pi\pi}(s-2M_\pi^2)+2g_3^{\pi\pi}(u^2+t^2-2s^2-4M_\pi^2(u+t-2s)) \ , \end{aligned} \qquad (4.72)$$

where,

$$\begin{aligned} \lambda_A &= g_3^{S_\eta\pi\pi} - g_1^{S_\eta\pi\pi} \ , \\ \lambda_B &= -M_\pi^2 \, g_3^{S_\eta\pi\pi} + g_4^{S_\eta\pi\pi} \ . \end{aligned} \qquad (4.73)$$

The first curly bracket is from ρ graph, the second curly bracket is from S_η and $S_{\eta'}$ exchange and remaining terms are from contact term. For s-wave only T^0 and T^2 are nonzero and for p-wave only T^1 is nonzero, due to Bose statistics. The s-wave scattering lengths and effective ranges which are obtained from (64), (65), (67), (71) and (72) are as follows (notations: a_{lT}) ,

$$8\pi M_\pi a_{00} = M_\pi^2 (4(g_1^{\rho\pi\pi})^2/M_\rho^2 + 2g_2^{\pi\pi}) + 10g_1^{\pi\pi} + (3\lambda_c\lambda_c' + 2\lambda_B^2/M_{S_\eta}^2 + S_\eta \leftrightarrow S_{\eta'}) , \quad (4.74)$$

$$4\pi M_\pi a_{02} = -M_\pi^2((g_1^{\rho\pi\pi})^2/M_\rho^2 + g_2^{\pi\pi}) + 2g_1^{\pi\pi} + (\lambda_B^2/M_{S_\eta}^2 + S_\eta \leftrightarrow S_{\eta'}) , \quad (4.75)$$

$$2\pi M_\pi((a_{00})^2 r_{00} - a_{00}/M_\pi^2) = -E_1 + (E_2 + S_\eta \leftrightarrow S_{\eta'}) - (6\lambda_c'(\lambda_B + \lambda_c') + S_\eta \leftrightarrow S_{\eta'}) , \quad (4.76)$$

$$4\pi M_\pi((a_{02})^2 r_{02} - a_{02}/M_\pi^2) = -E_1 + (E_2 + S_\eta \leftrightarrow S_{\eta'}), \quad (4.77)$$

where,

$$\lambda_c = 2M_\pi^2 \lambda_A + \lambda_B ,$$

$$\lambda_c' = \lambda_c/(M_{S_\eta}^2 - 4M_\pi^2),$$

$$E_1 = 3(g_1^{\rho\pi\pi})^2/M_\rho^2 + 2g_2^{\pi\pi} - 4M_\pi^2(g_1^{\rho\pi\pi}\lambda_D + 3M_\rho^4 g_3^{\pi\pi\rho}/2)/M_\rho^4 ,$$

$$\lambda_D = g_1^{\rho\pi\pi} + 2M_\rho^2 g_3^{\rho\pi\pi} ,$$

$$E_2 = 2\lambda_B(\lambda_A + \lambda_B/M_{S_\eta}^2)/M_{S_\eta}^2 . \quad (4.78)$$

The p-wave scattering length and effective range are,

$$24\pi M_\pi a_{11}^1 = (2g_2^{\pi\pi} + 8g_3^{\pi\pi}M_\pi^2 + (g_1^{\rho\pi\pi})^2/M_\rho^2) + 2(E_2/M_{S_\eta}^2 + S_\eta \leftrightarrow S_{\eta'}) + 2\lambda_E\lambda_E' + 4M_\pi^2 g_1^{\rho\pi\pi}\lambda_D/M_\rho^2 , \quad (4.79)$$

and,

$$3\pi M_\pi((a_{11})^2 r_{12} - a_{11}/M_\pi^2) = -2g_3^{\pi\pi} + 4M_\pi^2(g_1^{\rho\pi\pi} - M_\rho^2 g_3^{\rho\pi\pi})^2/M_\rho^6 - 4g_1^{\rho\pi\pi}\lambda_D/M_\rho^4 - 2\lambda_E'(\lambda_E' + 2g_3^{\rho\pi\pi}) + ((2\lambda_B + M_{S_\eta}^2\lambda_A)^2/M_{S_\eta}^6 + S_\eta \leftrightarrow S_{\eta'}), \quad (4.80)$$

where,

$$\lambda_E = g_1^{\rho\pi\pi} + 4M_\pi^2 \, g_3^{\rho\pi\pi} \quad ,$$

$$\lambda_E' = \lambda_E/(M_\rho^2 - 4M_\pi^2) \quad . \tag{4.81}$$

Let us consider the first term in a_{00} in (74) using (3.19), (3.20), (3.24), (5.11) and appropriate coupling constants from appendix, it can be written as,

$$(2M_\pi^2/F_\pi^2)\ (c/(m_0^2 + c))\,(8-c/m_0^2) = 7M_\pi^2/F_\pi^2 \ ,$$

$$\text{if } m_0^2 = c \tag{4.82}$$

where,

$$c = g^2(f_0 + f_8/\sqrt{3})^2 \ .$$

This is the soft pion result obtained by Weinberg[44]. The condition used is nothing but KSRF[45] ($F_\rho^2 = 2F_\pi^2 M_\rho^2$, discussed in next chapter). Similar manipulation on first term of (75) gives the soft pion results e.g.,

$$16\pi a_{02} = - M_\pi F_\pi^{-2} \quad . \tag{4.83}$$

The leading term in the expansion of the first bracket in (79) in powers of M_π^2 is the current algebra result i.e. $24\pi M_\pi a_{11} = F_\pi^{-2}$. Thus, in our model, one obtains soft pion result, for s-wave, only when the scalar meson contribution and contact term contribution proportional to $g_1^{\pi\pi}$ are neglected.

Experimentally, there is little knowledge on π-π scattering. The analysis of experimental data on $K^+ \to \pi^+\pi^- e^+\nu$ decay gives[46],

$$M_\pi a_{00} = 0.6^{+.6}_{-.5} \tag{4.84}$$

Table IV: s-and p-Wave π-π Scattering Lengths and Effective Ranges, in Units of M_{π}^{-1}, for Some Value of x (Notation a_{JI})

x	a_{00}	a_{02}	$a_{11}(x\cdot 10^{-2})$	r_{00}	r_{02}	r_{11}
0.90	-0.66	-0.39	3.30	- -2.6	-1.3	23
1.00	-0.20	-0.23	2.89	- 19.6	-0.37	27
1.05	0.05	-0.14	2.66	-250.1	4.6	30
1.10	0.[illegible]5	-0.04	2.40	- 3.94	109.0	34
1.15	0.59	0.06	2.13	- 0.48	96.7	40
1.20	0.88	0.16	1.82	0.074	16.7	49
1.25	1.19	0.27	1.49	0.20	7.7	65

whereas another analysis[47] gives,

$$M_\pi\, a_{00} = 0.71 \pm 0.37 \quad . \tag{4.85}$$

The measurement[48] of total $\pi - \pi$ cross-section and angular distribution of pions give,

$$M_\pi |a_{00}| = 0.52 \pm 0.06, \quad M_\pi |a_{02}| = 0.17 \pm 0.02. \tag{4.86}$$

The absolute sign of a_{00} remain undetermined. The study of inelastic scattering $N\pi \to N\pi\pi$ although do not measure a_{00} and a_{02} seperately but their ratio can be determined. These measurements give[49],

$$a_{00}/a_{02} = -3.2 \pm 0.1 \quad , \tag{4.87}$$

and[50]

$$a_{00}/a_{02} = -3.1 \pm 1.1 \quad , \tag{4.88}$$

The a_{0I} are function of x only whereas r_{1I} and a_{11} are function of x and δ both; however, the variation with δ is small. For those values of x, for which $M_\pi\, a_{00}$ varies from a small negative value to 1 and for $\delta = 0$ the scattering lengths and effective ranges are given in the Table IV. We see that the variation with x is quite significant. The contribution of first bracket in (74) to $M_\pi\, a_{00}$ is 0.175. Apart from this, there are three other contributions, $a_{S\eta}$ from S_η intermediate state, $a_{S\eta'}$ from $S_{\eta'}$ state and a_{c0} proportional to $g_1^{\pi\pi}$. At $x = +0.9$ a_{c0} is dominating ($M_\pi\, a_{c0} = -7.572$, $M_\pi\, a_{S\eta} = 4.985$ and $M_\pi\, a_{S\eta'} = 2.211$) so a_{00} is negative. Now as x increases a_{c0} and $a_{S\eta}$ increase and $a_{S\eta'}$ decreases. Since rate of variation of $a_{S\eta}$ is faster than a_{c0}, around $x \sim 1.07$ these

contributions, although large individually, cancell out giving current algebra result.[44] Such cancellations have also been observed by authors of ref. (51) who, however, have only one scalar meson at 750 MeV or 900 MeV. As x increases further $a_{S\eta}$ dominates giving large a_{00}. Similar is the variation in a_{02}. In the p-wave scattering length $M_\pi^3 \, a_{11}$ the ρ contribution is + 0.055, contact term contribution is - 0.02 and rest is from scalar meson. Therefore, for the range of x favoured by s-wave scattering lengths, we see that dominant contribution to p-wave scattering lengths come from ρ intermediate state which is in agreement with the usually assumed ρ dominance of p-wave $\pi - \pi$ scattering.

Many authors have calculated $\pi - \pi$ scattering lengths. The answer for $M_\pi \, a_{00}$ varies from -1 to 2 depending upon the technique used. Weinberg[44] employed current algebra and soft pion techniques. He further assumed that $\pi - \pi$ amplitude is atmost linear in s, t and u and the equal time commutator of fourth component of axial vector current with divergence of axial vector current (σ - commutator) is proportional to I = 0 scalar field. This way, he obtained $M_\pi \, a_{00} = 0.2$ and $a_{00}/a_{02} = -3.5$. Khuri[52] relaxed the linearity assumption and obtained essentially the same result. Illiopoulos[53] expanded amplitude in the variable $\sqrt{1-s^2}$, $\sqrt{1-t^2}$ and $\sqrt{1-u^2}$, imposed unitarity at threshold then besides getting Weinberg results obtained some other solutions like $a_{00} \sim - 2.45$, $a_{02} \sim -0.11$

and $a_{00} \sim 0$, $a_{02} \sim -2.3$. Bars[54] allowed an $I = 2$ component in σ-commutator and obtained essentially the Weinberg results. Since these calculations give scattering lengths at a unphysical points, (soft pions), attempts were made to extrapolate these to mass shell of pions. It is shown by Sucher and Woo[55] that extrapolation in general is not unique, for example if one imposes unitarity on the Weinberg series expansion one can obtain large $M_\pi a_{00} \sim 1.9$.

The dispersion relation approach to $\pi - \pi$ leads to different results depending on the assumption made. Fulco and Wong[56] after imposing some asymptotic limits obtained $M_\pi a_{00} = 0.8$. Paver and Verzegnassi[57] combining analyticity, unitarity and crossing symmetry with dispersion relations obtained inequalities like $0 \leq M_\pi a_{00} \leq 0.89$, $-0.15 \leq M_\pi a_{02} \leq 0.77$ and $M_\pi^3 a_{11} > 0.07$. However, Gore[58] with the same technique but with different saturation procedure obtained $M_\pi a_{00} = -0.69$, $M_\pi a_{02} = -0.37$ and $M_\pi^3 a_{12} = 0.028$. Patil[59] from some other considerations obtained $M_\pi a_{00} = 1.2$.

The effective Lagrangian calculations[60] predict small scattering lengths near about the Weinbergs value.

(ii) <u>$K - \pi$ Scattering</u>:

For $K - \pi$ scattering,

$$K(k_c) + \pi(p_a) \rightarrow K(k_d) + \pi(p_b) \quad , \tag{4.89}$$

the amplitude T(s, t, u) is written as,

$$T(s,\ t,\ u) \;=\; \delta_{ab}\, T^{+}(s,\ t,\ u) + \frac{1}{2}\,[\tau_b,\ \tau_a]_-\ T^{-}(s,t,u), \tag{4.90}$$

where 'a' and 'b' are the isospin indices of the initial and final pion. The I = 3/2, and 1/2 s-channel amplitudes are,

$$\begin{aligned} T^{3/2} &= T^{+} - T^{-} \quad , \\ T^{1/2} &= T^{+} + 2T^{-} \quad . \end{aligned} \tag{4.91}$$

In K – π scattering K^*, S_K poles appear in s-and u-channels, ρ, S_η and $S_{\eta'}$ poles appear in the t-channel and finally there is contribution from the contact term. Using the effective Lagrangian for various vertices one can calculate the contribution from various diagrams shown in Fig. 2. The result is,

$$\begin{aligned} T^{+}(s,\ t,\ u) &= T^{+}_{K^*}(s,t,u) + T^{+}_{S_K}(s,\ t,\ u) \\ &\quad + T_{s}(s,\ t,\ u) + T^{+}_{c}(s,\ t,\ u)\ , \\ T^{-}(s,\ t,\ u) &= T^{-}_{K^*}(s,t,u) + T^{-}_{S_K}(s,\ t,\ u) \\ &\quad + T_{\rho}(s,\ t,\ u) + T^{-}_{c}(s,\ t,\ u)\ , \end{aligned} \tag{4.92}$$

where,

$$\begin{aligned} T^{\pm}_{K^*}(s,\ t,\ u) &= T_{K^*}(s,\ t,\ u) \pm s \leftrightarrow u\ , \\ T^{\pm}_{S_K}(s,\ t,\ u) &= T_{S_K}(s,\ t,\ u) \pm s \leftrightarrow u\ , \\ (M^2_{K^*}-s)\ T_{K^*}(s,\ t,\ u) &= (2M^2_{\pi} - t)\ g_1^2/2 + (2M^2_K - t)g_2^2/2 \\ &\quad + (M^2_{\pi}+M^2_K - u)\, g_1 g_2 - (s_2 g_1^{K^*K\pi} + s_1 g_2^{K^*K\pi})^2/4M^2_{K^*}\ , \end{aligned}$$

$$s_1 = s - M_\pi^2 + M_K^2 , \quad s_2 = s + M_\pi^2 - M_K^2 ,$$

$$g_1 = g_1^{K^*K\pi} + s_1 g_3^{K^*K\pi}, \quad g_2 = g_1^{K^*K\pi} - s_2 g_3^{K^*K\pi} ,$$

$$4(M_{S_K}^2 - s)\, T_{S_K}(s,\ t,\ u) = s_1 g_1^{S_K K\pi} + s_2 g_2^{S_K K\pi}$$

$$-(s - M_\pi^2 - M_K^2) g_3^{S_K K\pi} - 2 g_4^{S_K K\pi})^2 ,$$

$$(M_{S_\eta}^2 - t)\, T_S\,(s,t,u) = (-t g_1^{S_\eta KK}/2 + (t/2 - M_K^2) g_3^{S_\eta KK} + g_4^{S_\eta KK})$$

$$x(-t g_1^{S_\eta \pi\pi}/2 + (t/2 - M_\pi^2) g_3^{S_\eta \pi\pi} + g_4^{S_\eta \pi\pi}),$$

$$(M_\rho^2 - t)\, T_\rho\,(s,\ t,\ u) = (u-s)(g_1^{\rho KK} + t g_3^{\rho KK}/2)(g_1^{\rho\pi\pi} + t g_3^{\rho\pi\pi}/2),$$

$$T_c^+\,(s,\ t,\ u) = g_1^{K\pi} + 2 g_2^{K\pi}(2M_\pi^2 + 2M_K^2 - s - u)$$

$$+ 2 g_3^{K\pi}\,(t - 2M_\pi^2) + 2 g_4^{K\pi}(t - 2M_K^2)$$

$$+ g_7^{K\pi}\,((u - M_\pi^2 - M_K^2)^2 + (s - M_\pi^2 - M_K^2)^2 - (t - 2M_\pi^2)(t - 2M_K^2)),$$

$$T_c^-\,(s,\ t,\ u) = g_6^{K\pi}((u - M_K^2 - M_\pi^2) - (s - M_K^2 - M_\pi^2)^2)/16 .$$

The s wave scattering lengths $a_0^\pm$ and effective ranges $r_0^\pm$ are,

$$8\pi(M_\pi + M_K)\, a_0^\pm = a_{K^*}^\pm + a_{S_K}^\pm + R^\pm , \tag{4.93}$$

and,

$$-4\pi(M_\pi + M_K)((a_0^\pm)^2 r_0^\pm - a_0^\pm/M_K M_\pi)$$

$$= (r_{K^*}^\pm + d_\pm a_{K^*}^\pm)/(M_{K^*}^2 - (M_K \pm M_\pi)^2) + E^\pm$$

$$+ (r_{S_K}^\pm + d_\pm a_{S_K}^\pm)/(M_{S_K}^2 - (M_K \pm M_\pi)^2) \tag{4.94}$$

where,

$$d_+ = M_\pi/M_K + M_K/M_\pi + 2 ,$$

$$d_- = -M_\pi/M_K - M_K/M_\pi ,$$

$$M^2_{K^*}a^{\pm}_{K^*} = (M_\pi g_1^{K^*K\pi} + M_K g_2^{K^*K\pi})^2 \pm (M_\pi g_1^{K^*K\pi} - M_K g_2^{K^*K\pi})^2 ,$$

$$a^{\pm}_{S_K} = (l_1^+)^2/(M^2_{S_K} - (M_K+M_\pi)^2) \pm (l_1^-)^2/(M^2_{S_K} - (M_K-M_\pi)^2) ,$$

$$l_1^{\pm} = l_2 + (M_K \pm M_\pi)^2\, l_3 ,$$

$$2l_2 = 2g_4^{S_K K\pi} + (M^2_\pi - M^2_K)g_1^{S_K K\pi} + (M^2_K - M^2_\pi)g_2^{S_K K\pi} - (M^2_K + M^2_\pi)\, g_3^{S_K K\pi} ,$$

$$l_3 = g_3^{S_K K\pi} - g_1^{S_K K\pi} - g_2^{S_K K\pi} ,$$

$$R^+ = (\lambda_B \lambda_G/M^2_{S_\eta} + S_\eta \leftrightarrow S_{\eta'}) + g_1^{K\pi} - 4(M^2_\pi g_3^{K\pi} + M^2_K g_4^{K\pi}) ,$$

$$\lambda_G = -M^2_K\, g_3^{S_\eta KK} + g_4^{S_\eta KK} ,$$

$$R^- = 4M^2_K\, g_1^{\rho KK}\, g_1^{\rho\pi\pi}/M^2_\rho ,$$

$$r^{\pm}_{K^*} = l_4^{\pm} + l_5^{\pm} + 2l_7^{\pm}\,(M^2_\pi l_4^{\pm} + M^2_K l_5^{\pm} \pm 2M_\pi M_K(l_5^{\pm} - l_4^{\pm})) + d_{\pm} l_4^{\pm} l_5^{\pm} + l_6^{\pm}\, l_8^{\pm}/2M^2_{K^*} ,$$

$$l_4^{\pm} = g_1^{K^*K\pi} + 2M_K\,(M_K \pm M_\pi)\, g_3^{K^*K\pi} ,$$

$$l_5^{\pm} = g_2^{K^*K\pi} - 2M_\pi\,(M_\pi \pm M_K)\, g_3^{K^*K\pi} ,$$

$$l_6^{\pm} = 2(M_K \pm M_\pi)(M_K g_2^{K^*K\pi} \pm M_\pi g_1^{K^*K\pi}) ,,$$

$$l_7^{\pm} = d_{\pm}\, g_3^{K^*K\pi} ,$$

$$l_8^{\pm} = d_{\pm}\,(g_1^{K^*K\pi} + g_2^{K^*K\pi}) ,$$

$$r^{\pm}_{S_K} = 2l^{\pm}_1 \, d_{\pm} \, l_3 \quad ,$$

$$E^{+} = ((-2\lambda_B\lambda_G/M^2_{S_\eta} + \lambda_B\lambda_H + \lambda_A\lambda_G)/M^2_{S_\eta} + S_\eta \leftrightarrow S_{\eta'})$$
$$- 4g_2^{K\pi} - 4(g_3^{K\pi} + g_4^{K\pi}) + 8M_\pi M_K \, g_7^{K\pi} \quad ,$$

$$\lambda_G = g_3^{S_\eta KK} - g_1^{S_\eta KK} \quad ,$$

$$E^{-} = -4M_\pi M_K \, g_6^{K\pi}/3 \quad .$$

The λ_A and λ_B are given by (73).

The K-π s-wave scattering lengths and effective ranges (in units of M_π^{-1}) for some values of x ($\delta = 0$) are as follows:

x	$a_0^{3/2}$	$a_0^{1/2}$	$r_0^{3/2}$	$r_0^{1/2}$	a_0^-	a_0^+
1.0	-0.034	-0.207	-85	5.8	-0.057	-0.091
1.1	-0.60	-0.233	-29	4.5	-0.057	-0.117
1.2	-0.088	-0.261	-14	3.4	-0.058	-0.145

As in π-π case, in s-wave K-π scattering lengths the scalar mesons contributions are large. The current algebra technique[44] and effective Lagrangian methods[61] give $M_\pi^{-1} \, a_0^{3/2} = -0.066$ and $M_\pi^{-1} \, a_0^{1/2} = 0.13$. Some calculations[62] which obtain information only about $M_\pi \, a_0^{-1}(M_\pi(a_0^{1/2}-a_0^{3/2})/3)$ give it between 0.066 to 0.136. Our results differ from these works because of large scalar contributions about which none of the above calculations obtain any information.

(iii) K-K Scattering:

In K-K scattering,

$$K(k_a) + K(k_b) \rightarrow K(p_c) + K(p_d), \tag{4.95}$$

we have two isospin amplitudes T^1 and T^0 corresponding to $I = 1, 0$ respectively. The diagrams contributing to the process are those shown in Fig. 2; ρ, ϕ, S_η, $S_{\eta'}$ and S_π poles and the contact term. As before, using effective Lagrangian for various vertices we obtain,

$$T^1_{KK}(s,\ t,\ u) \ = \ T^1_\rho + T^1_\phi + T^1_{S_\pi} + T^1_S + T_c \ , \tag{4.96}$$

and

$$T^0_{KK}(s,\ t,\ u) \ = \ T^0_\rho + T^0_\phi + T^0_{S_\pi} + T^0_S \ , \tag{4.97}$$

where,

$$T^1_\rho = T_\rho(s,t,u)+u \quad t; \ T^0_\rho = -3T_\rho(s,t,u)+u \leftrightarrow t \ ,$$

$$T^1_\phi = T_\phi(s,t,u)+u \leftrightarrow t; \ T^0_\phi = T_\phi(s,t,u) - u \leftrightarrow t \ ,$$

$$T^1_{S_\pi} = T_{S_\pi}(s,t,u)+u \leftrightarrow t; T^0_{S_\pi} = 3T_{S_\pi}(s,t,u) - u \leftrightarrow t \ ,$$

$$T^1_S = T_S(s,t,u)+t \leftrightarrow u; \ T^0_S = -T_S(s,t,u) + t \leftrightarrow u \ ,$$

$$T_\rho(s,t,u) = (g_1^{\rho KK} + ug_3^{\rho KK})^2(t-s)/(M^2_\rho - u) \ ,$$

$$T_\phi(s,t,u) = (g_1^{\phi KK} + ug_3^{\phi KK})^2(t-s)/(M^2_\phi - u) \ ,$$

$$T_S(s,t,u) = (-ug_1^{S_\eta KK}/2+(u/2-M_K^2)g_3^{S_\eta KK}+g_4^{S_\eta KK})^2/(M^2_{S_\pi}-u)+S_\eta \leftrightarrow S_{\eta'} \ ,$$

$$T_{S_\pi}(s,t,u) = (-ug_1^{S_\pi KK}/2+(u/2-M_K^2)g_3^{S_\pi KK}+g_4^{S_\pi KK})^2/(M^2_{S_\pi}-u) \ ,$$

$$T_c(s,t,u) = 4g_1^{KK} - 4g_1^{KK}(2S+u+t-8M_K^2) + 2g_3^{KK}(2s^2-t^2-u^2-4M_K^2(2s-u-t)).$$

The only nonzero s-wave amplitude is T^1. We obtain for $a_0^1(KK)$,

$$16\pi M_K a_0^1(KK) = -4M_K^2(g_1^{\rho KK})^2/M_\rho^2 - 4M_K^2(g_1^{\phi KK})^2/M_\phi^2 + w_1^2/M_{S_\pi}^2 + w_2^2/M_{S_\eta}^2 + 4g_4^{KK} , \qquad (4.98)$$

and the effective range is given by,

$$-4\pi M_K((a_0^1(KK))^2 r_0^1 - a_0^1(KK)/M_K^2)$$
$$= -g_1^{\rho KK}\,(g_1^{\rho KK}(3-4M_K^2/M_\rho^2)-8M_K^2 g_3^{\rho KK})/M_\rho^2$$
$$-g_1^{\phi KK}\,(g_1^{\phi KK}(3-4M_K^2/M_\phi^2)-8M_K^2 g_3^{\phi KK})/M_\phi^2$$
$$+w_1(g_1^{S_\pi KK} - g_3^{S_\pi KK} - w_1/M_{S_\pi}^2)/M_{S_\pi}^2$$
$$+w_2(g_1^{S_\eta KK} - g_3^{S_\eta KK} - w_2/M_{S_\eta}^2)/M_{S_\eta}^2 + S_\eta \leftrightarrow S_{\eta'}$$
$$-4g_2^{KK} + 8g_3^{KK} M_K^2 \quad . \qquad (4.99)$$

where,

$$w_1 = g_4^{S_\pi KK} - M_K^2\, g_3^{S_\pi KK} , \quad w_2 = g_4^{S_\eta KK} - M_K^2 g_3^{S_\eta KK} .$$

The K-K s-wave parameters for some values of x and $\delta = 0$ are as follows:

x	$M_\pi^{-1}a_0^1(KK)$	$M_\pi^{-1}r_0^1(KK)$
1.0	0.90	4.2
1.1	0.92	4.2
1.2	0.94	4.2

In this case $\rho + \phi$ and S_π contributions to $M_\pi^{-1}\ a_0^1$ are -0.11 and 0.34 respectively. Again scalar contributions dominate. For $M_\pi^{-1}a_0^1$ the effective Lagrangian approach give[61] -0.16.

From the discussion on scattering lengths, we note that in all the cases scalar mesons are playing an important role.

CHAPTER V

CURRENTS AND MESON FORM FACTORS

In this chapter, we obtain the vector and axial vector currents and their divergences. The currents responsible for weak decays of hadrons are then used to calculate decay constants of mesons, K_{13} and K_{14} form factors.

5-1 Vector and Axial Vector Currents:

The infinitesimal SU(3) transformations of the fields π_i, σ_i, y^i_μ and z^i_μ, as given by (2.17), (2.19), (2.48), (2.50) and (3.3) are,

$$
\begin{aligned}
\delta_V \sigma_i &= - f_{ijk}\, \epsilon^V_j \sigma_k \,, \quad \delta_V \pi_i = - f_{ijk}\, \epsilon^V_j\, \pi_k \,, \\
\delta_V y^i_\mu &= - f_{ijk}\, \epsilon^V_j\, y^k_\mu - \partial_\mu \epsilon^V_i / g \,, \\
\delta_V z^i_\mu &= - f_{ijk}\, \epsilon^V_j\, z^k_\mu \,,
\end{aligned}
\tag{5.1}
$$

whereas under the axial SU(3), transformations are,

$$
\begin{aligned}
\delta_A \sigma_i &= d_{ijk}\, \epsilon^A_j\, \pi_k \,, \quad \delta_A \pi_i = - d_{ijk}\, \epsilon^A_j\, \sigma_k \\
\delta_A y^i_\mu &= -f_{ijk}\, \epsilon^A_j\, y^k_\mu \,, \\
\delta_A z^i_\mu &= -f_{ijk}\, \epsilon^A_j\, z^k_\mu - \partial_\mu \epsilon^A_i / g \,.
\end{aligned}
\tag{5.2}
$$

When the symmetry is broken (1) and (2) no longer give the transformation laws for the physical fields. The transformations for the physical fields s_i, v^i_μ and a^i_μ are obtained by substituting (3.5), (3.8), (3.10), (3.12), (3.14) and (3.25) in (1) and (2). This gives,

$$
\begin{aligned}
\delta_V s_i &= -f_{ijk}\, \epsilon^V_j\, Z^{-1/2}_{s_i} (Z^{1/2}_{s_k} s_k + f_k) \\
\delta_V \pi_i &= -f_{ijk}\, \epsilon^V_j\, \pi_k \\
Z^{1/2}_{V_i} \delta_V v^i_\mu &= -f_{ijk}\, \epsilon^V_j\, Z^{1/2}_{V_k} v^k_\mu - \epsilon^V_j (f_{ijk} \xi^s_{kl} - f_{jlk} \xi^s_{ik}) Z^{1/2}_{s_l} \partial_\mu s_l \\
&\quad - \partial_\mu \epsilon^V_j (\delta_{ij}/g - f_{ljm} \xi^s_{il} (Z^{1/2}_{s_m} s_m + f_m))
\end{aligned}
$$

$$Z_{A_i}^{1/2}\delta_V a_\mu^i = -f_{ijk}\epsilon_j^V Z_{A_k}^{1/2} a_\mu^k - \epsilon_j^V(f_{ijk}\xi_{kl}^p - f_{jlk}\xi_{ik}^p)\partial_\mu \pi_l + \partial_\mu \epsilon_j^V f_{ijk}\xi_{il}^k \pi_k .$$

$$\delta_A s_i = d_{ijk}\,\epsilon_j^A\, Z_{s_i}^{-1/2}\, \pi_k$$

$$\delta_A \pi_i = -d_{ijk}\,\epsilon_j^A\,(Z_{s_k}^{1/2}\, s_k + f_k)$$

$$Z_{V_i}^{1/2}\,\delta_A v_\mu^i = -f_{ijk}\,\epsilon_j^A\, Z_{A_k}^{1/2}\, a_\mu^k - d_{kjl}\,\xi_{ik}^s\,\partial_\mu \epsilon_j^A\,\pi_l$$
$$-\epsilon_j^A(f_{ijk}\,\xi_{kl}^p + d_{kjl}\,\xi_{ik}^s)\,\partial_\mu \pi_l$$

$$Z_{A_i}^{1/2}\delta_A a^i = -f_{ijk}\,\epsilon_j\, Z_{V_k}^{1/2}\, v_\mu^k - \epsilon_j^A(f_{ijk}\,\xi_{kl}^s - d_{kjl}\,\xi_{ik}^p) Z_{s_l}^{1/2}\partial_\mu s_l$$
$$-\partial_\mu \epsilon_j^A(\delta_{ij}/g - d_{kjl}\,\xi_{ik}^p\,(Z_{s_l}^{1/2} s_l + f_l)) \; . \quad (5.3)$$

In (3) still the unrenormalized pseudoscalar fields π_i is written. Using (3.10), (3.12) and (3.25), it can be expressed in terms of physical pseudoscalar fields.

The currents are calculated from the Lagrangian by using Noethers theorem. If the fields ψ_i undergo infinitesimal transformations of the type,

$$\psi_i(x) \to \psi_i(x) + C_{iak}\,\epsilon_a(x)\psi_k(x) \quad , \qquad (5.4)$$

where C_{iak} are the constants characteristic of the group and the representation to which ψ belongs, then the current according to Noether theorem is given by,

$$J_{\mu j}(x) = -\frac{\partial L}{\partial \partial_\mu \psi_i(x)}\; C_{ijk}\psi_k(x) \; . \qquad (5.5)$$

The vector currents are calculated from (3) and (5). These, after using field equations of vector and axial vector mesons,

are given by,

$$J^V_{\mu i} = -\frac{m_0^2}{g}\,(Z^{1/2}_{V_i}\, v^i_\mu + \xi^s_{ij}\, Z^{1/2}_{s_j}\, \partial_\mu s_j) + K^V_{\mu i}\ , \quad (5.6)$$

where,

$$K^V_{\mu i} = -\partial_\nu F^{i'}_{\mu\nu}/g - f_{jkl}\, \partial_\nu (\xi^s_{il}\, F^{j'}_{\mu\nu}(Z^{1/2}_{s_k} s_k + f_k)\ ,$$
$$+ \xi^p_{il}\, G^{j'}_{\mu\nu}\, \pi_k\,)\ ,$$

$$F^{i'}_{\mu\nu} = F^i_{\mu\nu} + hf_{ijk}\,(D_\mu \pi_j\, D_\nu \pi_k + D_\mu s_j\, D_\nu s_k)$$
$$+h_1(C_{1ij}\, F^j_{\mu\nu} + C_{2ij}\, G^j_{\mu\nu}) + h_2(C_{3i} + C_{4i})_{\mu\nu}\ ,$$

$$G^{i'}_{\mu\nu} = G^i_{\mu\nu} - hd_{ijk}\,(D_\mu \pi_j\, D_\nu s_k - D_\nu \pi_j\, D_\mu s_k)$$
$$+h_1(C_{1ij}\, G^j_{\mu\nu} + C_{2ij}\, F^j_{\mu\nu}) + h_2(C_{3i} - C_{4i})_{\mu\nu}\ ,$$

$$C_{1ij} = \frac{1}{2}\, d_{ijk}\, d_{klm}\, ((Z^{1/2}_{s_l}\, s_l + f_l)(Z^{1/2}_{s_m} s_m + f_m) + \pi_l\, \pi_m)\ ,$$

$$C_{2ij} = -2\, d_{ijk}\, f_{klm}\, \pi_l\, (Z^{1/2}_{s_m}\, s_m + f_m)\ ,$$

$$C_{3i\mu\nu} = C_{5il}\, (F_{\mu\nu} - G_{\mu\nu})_l\ ,$$

$$C_{4i\mu\nu} = C_{5il}\, (F_{\mu\nu} + G_{\mu\nu})_l\ ,$$

$$C_{5il} = (d_{ijk} + if_{ijk})(d_{klm} + if_{klm})(Z^{1/2}_{s_j}\, s_j + f_j - i\pi_j)\ ,$$
$$\times\ (Z^{1/2}_{s_m}\, s_m + f_m + i\pi_m)/4\ .$$

The axial vector currents are given by,

$$J^A_{\mu i} = -\frac{m_0^2}{g}\,(Z^{1/2}_{A_i}\, a^i_\mu + \xi^p_{ij}\, \partial_\mu \pi_j) + K^A_{\mu i}\ , \quad (5.7)$$

where,

$$K^{A}_{\mu i} = -\partial_{\nu} G^{i'}_{\mu\nu}/g - d_{ijk}\partial_{\nu}(\xi^{p}_{il}\; G^{j'}_{\mu\nu}\,(Z^{1/2}_{s_k}\; s_k + f_k) + \xi^{s}_{il}\; F^{j'}_{\mu\nu}\,\pi_k\,) \,.$$

The currents (6) and (7) without K_{μ} term are precisely those obtained from field current identity model of Lee-Weinberg and Zumino[64]. In the subsequent calculations, we shall neglect the K_{μ} terms in (6) and (7). This is justified because the linear terms in fields in (6) and (7) generate meson pole terms which dominate the current matrix elements at low energies.

5-2 <u>Meson Decay Constants</u>:

Defining the decay constant of the S_K meson through the relation,

$$\langle 0|J^{VK^+}_{\mu}|S_{K^+}, q\rangle = -F_{S_K}\, q_{\mu} \,, \tag{5.8}$$

we obtain,

$$F_{S_K} = \sqrt{3}\, f_8\, Z^{-1/2}_{S_K}/2 \,. \tag{5.9}$$

The decay constants for the pseudoscalar mesons are defined through the relation,

$$\langle 0|J^{A}_{\mu k}|p_k, q\rangle\rangle = -iF_{p_k}\, q_{\mu} \quad , \quad k = 1, \ldots, 8$$

$$\langle 0|J^{A}_{\mu 8}|\eta', q\rangle = -iF_{\eta'}\, q_{\mu} \,. \tag{5.10}$$

For convenience, we have defined F's in (8) and (10) with an extra negative sign with compared to the conventional definition.

The decay constants of pseudoscalar mesons as given by (7) and (10) are,

$$F_\pi = (f_0 + f_8/\sqrt{3})\ Z_\pi^{-1/2} ,$$
$$F_K = (f_0 - f_8/2\sqrt{3}) Z_K^{-1/2} ,$$
$$F_\eta = m_0\ Z_\eta^{-1/2}(Z_\eta - 1)^{1/2} \cos\theta_p/g ,$$
$$F_{\eta'} = F_\eta \tan\theta_p . \qquad (5.11)$$

In the current-current picture, the Lagrangian which governs the weak decay of hadrons is of the form,

$$L_W = G\ J_\mu^+(x)\ J_\mu(x)/\sqrt{2} , \qquad (5.12)$$

where G is the weak decay coupling constant which is determined from $\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$. Its value is,

$$GM^2_{proton} = 1.05 \times 10^{-5} . \qquad (5.13)$$

The current J_μ is of the form,

$$J_\mu(x) = J_\mu^h(x) + J_\mu^l(x) . \qquad (5.14)$$

The hadronic part $J_\mu^h(x)$ is further written as,

$$J_\mu^h = (J_\mu^{V(1-i2)} \cos\theta_V + J_\mu^{V(4-i5)} \sin\theta_V)$$
$$+(J_\mu^{A(1-i2)} \cos\theta_A + J_\mu^{A(4-i5)} \sin\theta_A) , \qquad (5.15)$$

where θ_V and θ_A are the Cabbibo angles for the vector and axial vector currents respectively. The decay rate for the process ,

$$\pi^\pm \rightarrow l^\pm + \nu , \qquad (5.16)$$

is given by,

$$\Gamma(\pi \to l+\nu) = \frac{G^2}{8\pi} |F_\pi \cos\theta_A|^2 M_\pi m_l^2 (1-m_l^2/M_\pi^2). \quad (5.17)$$

The decay rate for the process,

$$K^\pm \to l^\pm + \nu , \quad (5.18)$$

is obtained from (17) by replacing $F_\pi \cos\theta_A$ by $F_K \sin\theta_A$ and M_π by M_K. The ratio of mode decay of K and π is,

$$\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{M_K}{M_\pi} \frac{(1-m^2/M_K^2)^2}{(1-m^2/M_\pi^2)^2} | F_K \tan\theta_A/F_\pi|^2. \quad (5.19)$$

Now using the experimental rates[23],

$$\tau(\pi \to \mu\nu) = 2.603 \times 10^{-8} \text{ sec},$$

$$\tau(K \to \mu\nu) = 1.937 \times 10^{-8} \text{ sec}, \quad (5.20)$$

and (19), one obtains,

$$|F_K \tan\theta_A/F_\pi|^2 = 0.0775 . \quad (5.21)$$

Although F_π and F_K individually depend on g, because masses determine g f_0 and g f_8 , their ratio is independent of g. We have $F_K/F_\pi = 1.04$. Then (21) gives,

$$\tan\theta_A = 0.268 \text{ or } \theta_A = 15^\circ . \quad (5.22)$$

Once θ_A is known g can be determined from the experimental decay rate of $\pi \to \mu\nu$. The value of g implied by (11), (17) and (22) is,

$$g^2/4\pi = 1.16. \quad (5.23)$$

The various decay constants now have the following values,

$$
\begin{aligned}
F_\pi &= 91.3 \text{ MeV}, \\
F_K &= 94.7 \text{ MeV} \; ; \quad F_K/F_\pi = 1.04 \, , \\
F_\eta &= 93.8 \text{ MeV} \; ; \quad F_\eta/F_\pi = 1.03 \, , \\
F_{\eta'} &= -19.3 \text{ MeV} \; ; \quad F_{\eta'}/F_\pi = -0.21 \, , \\
F_{S_K} &= -12.2 \text{ MeV} \; ; \quad F_{S_K}/F_\pi = -0.13 \, .
\end{aligned}
\tag{5.24}
$$

The predicted F_K/F_π comes close to the SU(3) value i.e. 1. This ratio has been of considerable interest. A set of calculations using different techniques predict[65] $F_K \sim F_\pi$. Our value of F_{S_K}/F_π is much smaller than the value obtained by Weinberg and Glashow[13] (= -0.58), Gasiorowicz and Geffen[16] (= -0.4) and others[66-68]. The π, K and S_K masses, their decay constants and renormalization constants satisfy the Glashow - Weinberg[13] relations,

$$
\begin{aligned}
M_\pi^2 \, F_\pi \, Z_\pi^{-1/2} &= M_K^2 \, F_K \, Z_K^{-1/2} + M_{S_K}^2 \, F_{S_K} \, Z_{S_K}^{-1/2} \, , \\
F_\pi \, Z_\pi^{1/2} &= F_K \, Z_K^{1/2} + F_{S_K} \, Z_{S_K}^{1/2} \, .
\end{aligned}
\tag{5.25}
$$

On eliminating Z_{S_K} from these two sum rules, we obtain,

$$
F_{S_K}^2 \, M_{S_K}^2 = M_\pi^2 \, F_\pi^2 + M_K^2 \, F_K^2 - ((Z_\pi/Z_K)^{1/2} \, M_K^2 + (Z_K/Z_\pi)^{1/2} M_\pi^2) F_\pi F_K .
\tag{5.26}
$$

Eq. (26) gives following Weinberg - Glashow[13] sum rules,

(a) If $(Z_\pi/Z_K)^{1/2} > 0$, $F_\pi F_K > 0$

$$M_{S_K} \leq |M_\pi F_\pi - M_K F_K|/|F_{S_K}| \quad , \tag{5.27}$$

(b) If $(Z_\pi/Z_K)^{1/2} > 0$, $F_\pi F_K < 0$

$$M_{S_K} \geq |M_\pi F_\pi + M_K F_K|/|F_{S_K}| \quad . \tag{5.28}$$

The estimate of various quantities from our model gives for these inequalities $M_{S_K} \leq 2785$ MeV and $M_{S_K} \geq 4879$ MeV. In our case $F_\pi F_K > 0$ so the first inequality for M_{S_K} mass is obeyed. Weinberg and Glashow had for these inequalities $M_{S_K} \leq 670$ MeV and $M_{S_K} \geq 1100$ MeV. It is to be noted from (27) that a lower value of ξ (or what is equivalent a lower value of F_{S_K}/F_π) would give a higher value of the upper limit in the inequality (27). In the Glashow and Weinberg model F_{S_K}/F_π is of the order of -0.4 (our value is -0.13) also in some other models[66,68] it is large, therefore these models predict M_{S_K} below the $K - \pi$ threshold.

The matrix elements of currents between vacuum and one spin one meson states are defined in the usual way as,

$$\langle 0|J^V_{\mu i}|v^i, q\rangle = F_{V_i}\, \epsilon^V_\mu(q) \quad ,$$

and,

$$\langle 0|J^A_{\mu i}|a^i, q\rangle = F_{A_i}\, \epsilon^A_\mu(q) \quad , \tag{5.29}$$

where ϵ^V_μ and ϵ^A_μ are the polarisation vectors of vector and axial vector mesons respectively. Eqs. (6), (7) and (29) give,

$$F_\rho Z_\rho^{-1/2} = F_{K^*} Z_{K^*}^{-1/2} = F_\phi Z_\phi^{-1/2} = F_{A_1} Z_{A_1}^{-1/2}$$

$$= F_{K_A} Z_{K_A}^{-1/2} = F_E Z_E^{-1/2} = -m_0^2/g \;, \qquad (5.30)$$

Using (3.19), (3.21), (11) and (30), we see that the relation,

$$F_\pi^2 = F_\rho^2/M_\rho^2 - F_{A_1}^2/M_{A_1}^2 \;, \qquad (5.31)$$

is satisfied. This is the Weinberg first sum rule[34] in the single particle approximation i.e. when current matrix elements are dominated by spin one and spin zero mesons poles. For strange vector and axial vector currents, the relation analogous to (31) is,

$$F_K^2 - F_{S_K}^2 = F_{K^*}^2/M_{K^*}^2 - F_{K_A}^2/M_{K_A}^2 \;. \qquad (5.32)$$

For I = 0, Y = 0 currents the relation,

$$F_\eta^2 + F_{\eta'}^2 = F_\phi^2/M^2 - F_E^2/M_E^2 \;, \qquad (5.33)$$

is satisfied. Although in our model the first Weinberg sum rule is satisfied, the second sum rule is not (we have it only if $h_2 = 0$). The nonvalidity of second sum rule in the presence of invariant corresponding to h_2 is discussed by Mitter and Swank.[69] They demonstrate that certain matrix elements do not have requisite asymptotic behaviour which is required in Weinberg's proof.

Kawarabayashi and Suzuki[45] and Riazzuddin and Fayazuddin[45] derived a relation between F_ρ, M_ρ and F_π namely,

$$F_\rho^2 = 2M_\rho^2 F_\pi^2 \;. \qquad (5.34)$$

In the present model, we do not have this, however, if the condition,

$$m_0^2 = g^2(f_0 + f_8/\sqrt{3})^2 \quad , \tag{5.35}$$

is imposed then KSRF follows. Incidently, if $h_2 = 0$, this condition also give famous Weinberg relation $M_{A_1} = \sqrt{2}M_\rho$. The numerical estimate of various quantities from our model give,

$$2M_\rho^2 \; F_\pi^2/F_\rho^2 = 1.09 \quad . \tag{5.36}$$

The divergence of the vector and axial vector currents can be calculated from the Lagrangian with the Gell-Mann-Levy[2] prescription,

$$\partial_\mu J_\mu = - \delta L/\partial \epsilon \quad . \tag{5.37}$$

It gives,

$$\begin{aligned}
\partial_\mu J^V_{\mu k} &= f_{klm}\, b_k\, Z^{1/2}_{s_l}\, s_l = i\, F_{S_k}\, M^2_{S_k}\, s_k \, , \\
\partial_\mu J^A_{\mu k} &= -d_{klm}\, b_l\, Z^{1/2}_{p_m}\, p_m = -F_{p_k}\, M^2_{p_k}\, p_k \, , \; k=1 \ldots 7 \, , \\
\partial_\mu J^A_{\mu 8} &= -(\sqrt{2/3}\, b_0 - b_8/\sqrt{3})(Z^{1/2}_\eta \cos\psi\, p_8 + \sin\psi\, p_0) \, , \\
&\quad -(\sqrt{2/3}\, b_8\, (-Z^{1/2}_\eta \sin\psi\, p_8 + \cos\psi\, p_0) \quad .
\end{aligned} \tag{5.38}$$

These are the partial conservation equations for the currents. On the extreme right the usual form of the partial conservation equations is written. As expected contribution to current nonconservation comes from the explicit symmetry breaking term. Such partial conservation equations would hold in

any model in which the symmetry breaking term is a linear function of fields belonging to some linear representation of the basic symmetry group, as is the case with our L_5 term in (3.4).

5-3 K_{l3} - Form Factors:

Let us now consider the decay process,

$$K^{\pm}(k) \rightarrow \pi^0(k) + l^{\pm}(p_1) + \nu(p\ ) \quad ,$$

where l stands either for electron or muon. Although $K^0 \rightarrow \pi^- l^+ \nu$ is also allowed but it is sufficient to consider only K^+ decay because K^0 decay is related to K^+ decay through $\Delta I = 1/2$ rule. The matrix elements , defined by (4.3), for this process are,

$$\mathcal{M} = \frac{G}{\sqrt{2}} \frac{1}{(2\pi)^3} \sqrt{\frac{M_1 M_\nu}{E_1 E_\nu}}\, \bar{u}(p_1)\, \gamma_\mu (1-\gamma_5) v(p_\nu)$$
$$\langle \pi^0\ (p) | J^h_\mu | K^+(k) \rangle \quad . \tag{5.39}$$

The matrix elements of J^h_μ between π^0 and K^+ states, in general, are written as,

$$\langle \pi^0(p)|J^h_\mu|K^+(k)\rangle = \sqrt{2}\sin\theta_V \langle \pi^0(p)|J^{VK^+}_\mu|K^+(k)\rangle$$
$$= -\frac{1}{\sqrt{2}} \frac{\sin\theta_V}{(2\pi)^3\sqrt{4E_\pi E_k}} \left[P_{+\mu} f_+(q^2) + P_{-\mu} f_-(q^2)\right] , \tag{5.40}$$

where,

$$P_+ = k + p \quad ,$$
$$P_- = q = k - p \quad .$$

The form factors $f_+(q^2)$ and $f_-(q^2)$ are written such that in the SU(3) limit i.e. when all the pseudoscalar mesons are degenerate and strangeness changing vector current in conserved $f_+(0) = 1$ and $f_-(0) = 0$. In the linear approximation the form factors $f_\pm(q^2)$ are further written as,

$$f_\pm(q^2) = f_\pm(0) \left[1 - \lambda_\pm q^2/M_\pi^2 \right] . \tag{5.41}$$

Another parameter, ξ_{Kl3}, is defined as,

$$\xi_{Kl3} = f_-(0)/f_+(0) . \tag{5.42}$$

The decay width of $K^+ \to \pi^0 l^+ \nu$ as obtained from (4.36) and (39) is,

$$\Gamma(K^+ \to \pi^0 l^+ \nu) = \frac{G^2}{32\pi^2} \frac{\sin^2\theta_V}{M_K} \int dE_{\pi^0} \int dE_l$$

$$\times\left[(2p_l \cdot P_+ P_\nu \cdot P_+ - p_l \cdot p_\nu P_+ \cdot P_+)|f_+(q^2)|^2 \right.$$

$$\left. + 2M_l^2 p_\nu \cdot P_+ f_+(q^2) f_-(q^2) + M_l^2 p_l \cdot p_\nu |f_-(q^2)|^2 \right] . \tag{5.43}$$

The limit of integrations as given by (4.37) are,

$$E_{\pi^0}|_{max} = (M_{K^+}^2 + M_{\pi^0}^2 - M_l^2)/2M_{K^+} ,$$

$$E_{\pi^0}|_{min} = M_{\pi^0} ,$$

$$E_l \Big|_{min}^{max} = \left[(t+M_l^2)(M_{K^+}-E_{\pi^0}) \pm (t-M_l^2)(E_{\pi^0}^2 - M_{\pi^0}^2)^{1/2}\right]/2t , \tag{5.44}$$

where,

$$t = -P_-^2 = M_{K^+}^2 - 2E_{\pi^0} M_{K^+} + M_{\pi^0}^2 .$$

Using (41 - 43) and G from (13) the decay width of $K^+ \to \pi^0 e^+ \nu$ becomes,

$$\Gamma(K^+\to\pi^0 e^+\nu) = 77.5\ [f_+(0)\ \sin\theta_V]^2$$
$$\times[1+3.703\lambda_+ + 5.488\lambda_+^2]\ \times\ 10^6\ \sec^{-1}. \quad (5.45)$$

The branching ratio, R, of the muon mode to the electron mode is,

$$R = R_1/R_2 \quad , \quad (5.46)$$

where,

$$\begin{aligned} R_1 &= 0.6423 + 3.777\lambda_+ + 6.763\lambda_+^2 \\ &\quad + \xi_{K13}\ (0.1260 + 0.4747\ (\lambda_+ + \lambda_-) + 2.086\lambda_-^2) \\ &\quad + \xi_{K13}^2\ (0.0191 + 0.168\lambda_-^2 + 0.4102\lambda_-^2)\ , \\ R_2 &= 1 + 3.703\,\lambda_+ + 5.488\,\lambda_+^2\ . \end{aligned}$$

In the tree graph approximation contributions to K_{13} form factors come from the K^* and S_K intermediate states. The form factors defined by (40) are as follows:

$$f_+(q^2) = -\frac{F_{K^*}}{q^2+M_{K^*}^2}\ [g_1^{K^*K\pi} + g_2^{K^*K\pi} + 2q^2 g_3^{K^*K\pi}]\ ,$$

$$f_-(q^2) = -\frac{F_{K^*}}{M_{K^*}^2}\,\frac{M_K^2 - M_\pi^2}{q^2 + M_{K^*}^2}\ f_1 + \frac{f_2}{q^2 + M_{S_K}^2}$$
$$+\Big[\ \frac{F_{K^*}}{M_{K^*}^2}\ (g_1^{K^*K\pi} - g_2^{K^*K\pi}) + F_{S_K}(g_1^{S_K K\pi} + g_2^{S_K K\pi} - g_3^{S_K K\pi})\ \Big]\ , \quad (5.47)$$

where,

$$\begin{aligned} f_1 &= -g_1^{K^*K\pi} - g_2^{K^*K\pi} + 2M_{K^*}^2\ g_3^{K^*K\pi}\ , \\ f_2 &= 2g_4^{S_K K\pi} + (-M_K^2 + M_\pi^2 - M_{S_K}^2)\ g_1^{S_K K\pi} \\ &\quad + (M_K^2 - M_\pi^2 - M_{S_K}^2)\,g_2^{S_K K\pi} + (-M_K^2 - M_\pi^2 + M_{S_K}^2)\,g_3^{S_K K\pi}\ . \end{aligned}$$

Table V: K_{13} Parameters for Some Values of δ.

δ	0	0.4	0.8	1.2	Experimental values
λ_+	0.016	0.019	0.022	0.025	0.030 ± 0.007[a]
λ_-	0.0236	0.0234	0.0232	0.0231	
ξ_{K13}	−0.14	−0.18	−0.21	−0.24	
R	0.6426	0.6422	0.6418	0.6414	0.626 ± 0.019[a]

a. Ref. 70.

Using appropriate couplings from the appendix, (3.19), (9) and (30), one can see that the bracket term in $f_-(q^2)$ is identically equal to zero. From (41), (42) and (47), we obtain,

$$f_+(0) = -F_{K^*}(g_1^{K^*K\pi} + g_2^{K^*K\pi})/M^2_{K^*} = 0.992 \quad ,$$

$$\lambda_+ = M_\pi^2 \,(1+2F_{K^*}g_3^{K^*K\pi}/f_+(0))/M^2_{K^*} = 0.0158+0.0071\ \delta \quad ,$$

$$f_-(0) = -F_{K^*}(M_K^2-M_\pi^2)f_1/M^4_{K^*}+f_2/M^2_{S_K} = 0.117-0.081\ \delta \quad ,$$

$$\lambda_- = -M_\pi^2((F_{K^*}/M^6_{K^*})(M_K^2-M_\pi^2)\ f_1 + f_2/M^4_{S_K})/f_-(0)$$

$$= -(0.0029+0.0018\ \delta)/f_-(0) \quad , \qquad (5.48)$$

and $$\xi_{K13} = -0.118 - 0.082\ \delta \quad . \qquad (5.49)$$

For some values of δ, λ_+, λ_- and ξ_{K13} are given in Table V.

The experimental information on both K^+_{e3} and $K^+_{\mu 3}$ decay is available.[70] The study of pion energy spectrum in K^+_{e3} decay and the Dalitz plot analysis of K^+_{e3} gives[70],

$$\lambda_+ = 0.030 \pm 0.007 \quad , \qquad (5.50)$$

which corresponds in (48) to δ between 0.9 to 2.4. The quantity $f_+(0)$ is not directly accessible to the experiments because K_{e3} life time (45) involves one more free parameter i.e. θ_V. The fact that $f_+(0)$ is close to one is in agreement what one expects from Ademollo - Gatto theorem[71] i.e. $f_+(0)-1$ is of second order in symmetry breaking. The experimental

width[23] of K^+_{e3} ,

$$\Gamma(K^+_{e3}) = (3.93 \pm 0.06) \times 10^6 \ \mathrm{sec}^{-1} \quad , \qquad (5.51)$$

Eq. (45) and $\lambda_+ = 0$ give,

$$f_+(0) \ \sin\theta_V = 0.225 \quad . \qquad (5.52)$$

Now (21) and (52) together read,

$$\frac{F_K}{F_\pi \ f_+(0)} \ \frac{\tan\theta_A}{\sin\theta_V} = 1.24 \quad . \qquad (5.53)$$

In the frame work of single Cabbibo angle theory i.e. $\theta_A = \theta_V$, (53) can be written as,

$$\frac{F_K}{F_\pi \ F_+(0)} = 1.24 \ (\ 1 - 0.05/f_+^2 \ (0) \ )^{1/2} \ . \qquad (5.54)$$

Putting $f_+(0) = 1$ on r.h.s of (54), we get,

$$\frac{F_K}{F_\pi \ f_+(0)} = 1.20 \quad . \qquad (5.55)$$

Our model gives a rather small value ~ 1.04 for l.h.s. Now instead of taking $\theta_V = \theta_A$, we could as well have taken $f_+(0)$ from our model, $\lambda_+ = 0$ and calculated θ_V from (45) and (51). This yields,

$$\sin\theta_V = 0.23 \quad , \qquad (5.56)$$

which agrees well with the presently accepted value[16] $\sin\theta_V = 0.22$. It is clear that if F_K/F_π and $f_+(0)$ are close to one, $\theta_A \neq \theta_V$. However, if single Cabbibo angle theory is

assumed then, as is clear from (55), one has to invoke large symmetry breaking in either F_K/F_π or $f_+(0)$ or in both. In an attempt to reconcile (55) with $F_K/F_\pi \sim 1$ and $f_+(0) \sim 1$, Pandit and Rajsekharan[72] assumed that whereas single Cabbibo angle assumption is good, there is a violation of μ-e universality in the strangeness changing decays which accounts the discrepancy.

The experimental information on the $K^+_{\mu 3}$ decay, in principle, should provide information about ξ_{K13}, λ_+ and λ_-. There are two types of experimental information available from the studies on $K^+_{\mu 3}$ decay.[70] The experiments give for the branching ratio[70],

$$R = \Gamma(K^+_{\mu 3})/\Gamma(K^+_{e3}) = 0.626 \pm 0.019 \quad . \qquad (5.57)$$

It is seen from (46) that if one take λ_+ from K^+_{e3} analysis (50), then R depends on ξ_{K13} and λ_-, therefore one cannot determine ξ_{K13} and λ_- from R. However, the usual practice is to give ξ_{K13} for $\lambda_- = 0$. This gives $\xi_{K13} = -0.3$, whereas our predicted value it around -0.2. The muon polarisation measurements[70] in $K^+_{\mu 3}$ decay favours ξ_{K13} around -1.0.

In the last few years K_{13} form factors* have been discussed by several authors[73]. The form factors calculated by various techniques do not agree with each other in various details. In the dispersion theoretic approach one writes down

* In the rest of this section ξ stands for ξ_{K13} and f_+ for $f_+(0)$.

either unsubtracted dispersion relation (UDR) or once subtracted dispersion relation (OSDR) for $f_{\pm}(q^2)$. When these are saturated by K^* and S_K poles relations between K^* and S_K couplings and $f_{\pm}(q^2)$ are obtained. Since S_K mass and couplings are not known one makes further assumptions about these which introduce their own uncertainties. Mathur et al.[74] wrote UDR for $f_{\pm}(q^2)$ and worked in the limit of zero pion and kaon four momenta. Saturation of the sum rules by K^* and S_K (mass 725 MeV, width 10 MeV) yield $f_+ = 0.84$ and $\xi = -0.20$. Matusda and Oneda[75] essentially employed same technique but work in the limit $p^2 \rightarrow 0$ with scalar kaon at mass 725 MeV. They obtained $f_+ = 1.05$, $\xi = -0.16$. In the OSDR case ξ cannot be predicted due to the presence of the subtraction constant.

The application of current algebra and soft pion techniques[76] to K_{l3} decay give a relation between K_{l3} and K_{l2} form factors namely,

$$f_+ \;(M_\pi^2 = 0\;,\quad q^2 = -M_K^2) + f_- \;(M_\pi^2 = 0,\; q^2 = -M_K^2) = F_K/F_\pi \;. \tag{5.58}$$

This relation relates form factors at a point which is far away from the physical region. Although in our model we do not have this relation, an estimate from our model gives 1.040 for l.h.s. as compared to 1.037 for r.h.s. Some authors have combined dispersion theoretic approach with the soft pion result (58) to determine the subtraction constant. Pati and

Sebastian[77] assumed OSDR for $f_+(q^2)$, both UDR and OSDR separately for $f_-(q^2)$ and (K^*, S_K) saturation scheme. The UDR for $f_-(q^2)$ with S_K at 1100 MeV gives, $f_+ = 0.8$, $\lambda_{\pm} \sim 0.02$ and small negative ξ. These form factors depend considerably on the value of F_K/F_π and S_K mass. The results of OSDR for $f_-(q^2)$ are more or less same as in the previous case. Fuch[78] wrote UDR for $f_+(q^2)$ and OSDR for $f_-(q^2)$ in both variables q^2 and p^2 and worked in the limit $p^2 \to 0$. The saturation by K^* pole gives $f_+ = F_K/F_\pi$, $\lambda_{\pm} = 0.31$ and small negative ξ. Nith[79] wrote OSDR for $f_+(q^2) + f_-(q^2)$ and UDR for $f_+(q^2) - f_-(q^2)$. Saturation of these relations by K^* pole and taking $f_+ = 1$ gives $\xi = 0.06$, $\lambda_+ = 0.017$ and $\lambda_- = -0.09$.

The hard meson calculations, within the framework of current algebra, partial conservation of currents and pole dominance model of currents give a relation[13] namely,

$$f_+ = (F_\pi^2 + F_K^2 - F_{S_K}^2)/2F_\pi F_K \quad . \tag{5.59}$$

In our model this is satisfied. It is pointed out in ref.(16) that this result holds so long as f_+ does not depend upon the pion and kaon momenta. Chang and Leung[67] after imposing the condition $Z_K = Z_{S_K}$ (in our model $Z_K = 2.43\ Z_{S_K}$) and the smoothness condition that $S_K K\pi$ couplings do not depend on q^2 (in our model this is not valid)[80] obtained $f_+ = 0.92$, $\lambda_+ = 0.018$, $\lambda_- = -0.002$ and $\xi = -0.019$. Riazuddin and Sarker[81] imposed $F_{S_K} = 0$ and the SU(3) result $f_+ = 1$ to obtain

$\lambda_+ = 0.016$, $\lambda_- = -0.107$ and $\xi = 0.05$. Gerstein and Schnitzer[66] took $\Gamma(K^* \to K\pi)$, (53) and $\lambda_+ = 0.0238$ as inputs. The assumption $Z_K = Z_{S_K}$ gives, $M_{S_K} = 635$ MeV, $f_+ = 0.85$, $\lambda_- = -0.016$ and $\xi = -0.002$. If we take Z_K/Z_{S_K} from our model then ξ becomes -0.48.

In another method which gives form factors at mass shell one writes down the most general structure of three and four point functions involving spin zero and spin one mesons on which current algebra and partial conservation of current conditions are imposed. Pande[82] assumed a S_K meson at 1100 MeV of width 450 MeV and determined $f_+ = 0.95$, $\lambda_+ = 0.022$, $\lambda_- = 0.049$ and $\xi = -0.066$. Arnowitt et al[83] obtained with S_K mass at 1000 MeV, $f_+ = 0.85$, $\xi \approx 0$, $\lambda_+ = 0.025$ and $\lambda_- = -0.3$. These authors also discuss the possibility of large negative ξ with S_K mass at 1300 MeV. Our expression (47) is same as of these authors but coupling constants are different.

The effective Lagrangians have also been used in the study of K_{l3} form factors. Lee[84] constructed SU(3) X SU(3) nonlinear chiral Lagrangian without S_K. On imposing $f_+ = 1$ he obtained $\xi = 0.026$, $\lambda_+ = 0.018$ and $\lambda_- = -0.2$. A chiral nonlinear Lagrangian[68] which predict S_K at 630 MeV gives $f_+ = 0.96$, $\xi = -0.048$, $\lambda_+ = 0.022$ and $\xi\lambda_- = -0.002$.

Brandt and Prepatra[85] have argued that by modifying the definition of PCAC one can obtain ξ around -1, However,

Weinstein[86] has pointed out that the calculation of Brandt and Prepatra depends not on the modification of PCAC hypothesis but rather on a large SU(3) violation. Arnowitt et al.[87] have argued that so long explicit symmetry breaking transform as $(3, 3^*) + (3^*, 3)$ representation ξ will be around -0.2. In order to obtain, ξ around -1 one must include symmetry breaking transforming as $(8, 1) + (1, 8)$ representation of SU(3) X SU(3).

5.4 K_{14} Form Factors:

The K_{14} decay modes consistent with $\Delta Y/\Delta Q = 1$ rule are,

$$\begin{array}{llllllllllll} A. & K^{\pm} & \to & \pi^+ & + & \pi^- & + & 1^{\pm} & + & \nu & , \\ B. & K^{\pm} & \to & \pi^0 & + & \pi^0 & + & 1^{\pm} & + & \nu & , \\ C. & K^0 & \to & \pi^- & + & \pi^0 & + & 1^+ & + & \nu & . \end{array} \tag{5.60}$$

Let us consider the general process,

$$K_c(k) \to \pi_a(p_a) + \pi_b(p_b) + 1^+(p_1) + \nu(p_\nu) \quad , \tag{5.61}$$

where a, b, c denote the charge states of the respective particles. In this process, unlike K_{12} and K_{13}, both the vector and axial vector currents may, in general, contribute. The matrix element for this process is,

$$\mathcal{M} = \frac{G}{\sqrt{2}} \frac{1}{(2\pi)^3} \sqrt{\frac{M_1 M_\nu}{E_1 E_\nu}} \; \bar{u}_1 \gamma_\mu (1+\gamma_5) u_\nu$$
$$\times \sqrt{2} < \pi_a(p_a) \; \pi_b \; (p_b) | \sin \theta_V \; J_\mu^{VK^+} + \sin \theta_A \; J_\mu^{AK^+} | K_c(k) > \tag{5.62}$$

The matrix elements of the currents are written as,

$$< \pi_a(p_a)\ \pi_b(p_b)|\dot{J}_{\mu}^{AK^+}|K_c(k) > \ = \frac{i}{(2\pi)^{9/2}}\frac{1}{M_K}$$

$$\times \frac{1}{\sqrt{8E_aE_bE_c}} F_{\mu}^{cab}(k,\ p_a,\ p_b) \quad , \tag{5.63}$$

and,

$$< \pi_a(p_a)\ \pi_b(p_b)|J_{\mu}^{VK^+}|K_c(k) > \ = \frac{i}{M_K^3}\frac{1}{(2\pi)^{9/2}}$$

$$\times \frac{F_4^{cab}}{\sqrt{8E_aE_bE_c}} \epsilon_{\mu\nu\lambda\sigma} k_\nu p_{a\lambda} p_{b\sigma} . \tag{5.64}$$

Although, in principle, vector current contributes in our model its contribution is zero. We shall return to this point later. The form factor F_μ^{cab} can be further written as,

$$F_\mu^{cab} = \overline{\chi}_{curr}\,(\delta_{ab}\,F_\mu^+ + \frac{1}{2}\,[\tau_a, \tau_b]_-\,F_\mu^-)\chi_K \ , \tag{5.65}$$

where χ_K and χ_{curr} are the two component isospin wave function for the decaying particle and current respectively. The Lorentz decomposition of form factors $F_\mu^{\pm}$ is,

$$F_\mu^{\pm} = P_{+\mu}F_1^{\pm} + P_{-\mu}F_2^{\pm} + q_\mu F_3^{\pm} \ , \tag{5.66}$$

where,

$$P_+ = p_a + p_b \ ,$$

$$P_- = p_a - p_b \ ,$$

$$q = k - p_a - p_b \ .$$

The dimensionless form factors $F_{1,2,3,4}$ in general, are functions of the invariants $s = -P_+^2$, q^2 and $k.P_-$. From (65), we obtain for the matrix elements of the decays of (60),

$$F_\mu^{K^+\pi^+\pi^-} = F_\mu^+ + F_\mu^- ,$$
$$F_\mu^{K^+\pi^0\pi^0} = F_\mu^+/\sqrt{2} ,$$
$$F_\mu^{K^0\pi^-\pi^0} = -\sqrt{2}\, F_\mu^- , \qquad (5.67)$$

where the factor $1/\sqrt{2}$ in $F_\mu^{K^+\pi^0\pi^0}$ appears because of identity of two pions. From (67) it follows that the decay widths of various processes satisfy the sum rule,

$$\Gamma(K^+ \to \pi^+\pi^- l^+\nu) = 2\Gamma(K^+ \to \pi^0\pi^0 l^+\nu) + \frac{1}{2}\Gamma(K^0 \to \pi^-\pi^0 l^+\nu). \qquad (5.68)$$

Eq. (68) is a consequence of $\Delta I = 1/2$ rule which is built in our model because the currents are SU(3) octet currents. Bose statistics implies that,

$$F^+_{1,3}(k, p_a, p_b) = F^+_{1,3}(k, p_b, p_a) ,$$
$$F^+_2(k, p_a, p_b) = -F^+_2(k, p_b, p_a) ,$$
$$F^-_{1,3}(k, p_a, p_b) = -F^-_{1,3}(k, p_b, p_a) ,$$
$$F^-_2(k, p_a, p_b) = F^-_2(k, p_b, p_a) . \qquad (5.69)$$

In the problem only invariant which changes sign under $p_a \leftrightarrow p_b$ is $k.P_-$. Therefore F^+_2 and $F^-_{1,3}$ would be proportional to $k.P_-$. If $k.P_- = 0$, then we have the relations,

$$F^+_2 = F^-_{1,3} = 0 . \qquad (5.70)$$

It means, from (67), that to $K^+ \to \pi^0\pi^0 e^+\nu$ decay only $F^+_{1,3}$ to $K^0 \to \pi^-\pi^0 e^+\nu$ only F^-_2 and to $K^+ \to \pi^+\pi^- e^+\nu$ $F^+_{1,3}$ and F^-_2 contribute.

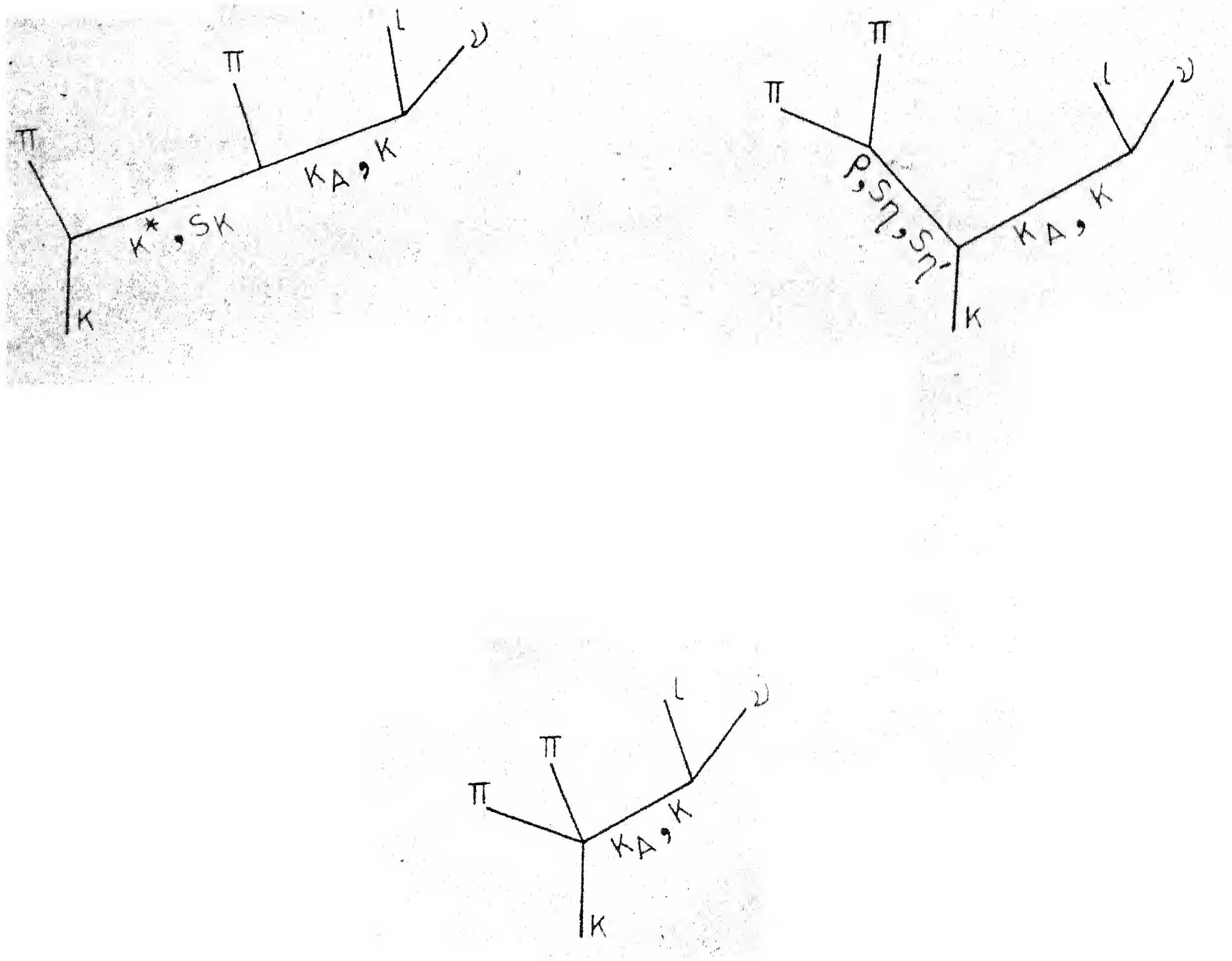

Fig. 3 Diagrams which contribute to the $K \to \pi\pi l\nu$ decay

In the tree graph approximation the diagrams which contribute to the matrix elements (63) are shown in the Fig.3. With the help of couplings given in the appendix the contribution of various diagrams can be calculated. These contributions are,

$$F^{+}_{\mu} = \frac{1}{2}\ \frac{M_K}{g}\ \frac{m_0^2}{q^2+M^2_{K_A}}\ [2F_{\mu}^{K_A}(k,\ p_a,\ p_b) + F_{\mu}^{K_A,S}\ (k,\ p_a, p_b)$$
$$+ (F_{\mu}^{K_A,\ K^*}(k,\ p_a,\ p_b) + F_{\mu}^{K_A,S_K}(k,\ p_a,\ p_b) + p_a \leftrightarrow p_b)]$$
$$+ \frac{1}{2}\ \frac{M_K\ F_K}{q^2+M_K^2}\ q_{\mu}[2F^{K}(k,\ p_a,\ p_b) + F^{K,S}\ (k,\ p_a,\ p_b)$$
$$+ (F^{K,K^*}(k,\ p_a,\ p_b) + F^{K,\ S_K}\ (k,\ p_a,\ p_b)+p_a \leftrightarrow p_b)]\ , \tag{5.71}$$

$$F^{-}_{\mu} = \frac{1}{2}\ \frac{M_K}{g}\ \frac{m_0^2}{q^2+M^2_{K_A}}\ [2\ F_{\mu}^{\prime K_A}\ (k,\ p_a,\ p_b) + F_{\mu}^{K_A,\rho}(k,\ p_a,\ p_b)$$
$$+ (F_{\mu}^{K_A,K^*}\ (k,\ p_a,\ p_b) + F_{\mu}^{K_A,S_K}(k,\ p_a,\ p_b) - p_a \leftrightarrow p_b)]$$
$$+ \frac{1}{2}\ \frac{M_K\ F_K}{q^2+M_K^2}\ q_{\mu}\,[2F^{\prime K}\ (k,\ p_a,\ p_b) + F^{K,\rho}(k,\ p_a,\ p_b)$$
$$+ (F^{K,K^*}(k,\ p_a,\ p_b) + F^{K,S_K}(k,p_a,p_b) - p_a \leftrightarrow p_b)]\ . \tag{5.72}$$

In writing (71) and (72) we have adopted following notations. The first superscript stands for the intermediate particle which connects a strong vertex to the weak vertex and second superscript represents the intermediate state connecting two strong vertices. The superscript s stand for $(S_{\eta},\ S_{\eta'})$ intermediate state. The various contributions are as follows:

$$F_{\mu}^{K_A}(k,\ p_a,\ p_b) = (P_{+\mu}+P_{+}\cdot qq_{\mu}/M_{K_A}^2)g_1^{K_AK\pi\pi}$$

$$-2(k_{\mu} + k.qq_{\mu}/M_{K_A}^2)g_2^{K_AK\pi\pi} + 2[k.p_b(p_{a\mu}+p_a\cdot qq_{\mu}/M_{K_A}^2)$$

$$+k.p_a(p_{b\mu}+p_b\cdot qq_{\mu}/M_{K_A}^2) - 2p_a.p_b\ (k_{\mu}+k.qq_{\mu}/M_{K_A}^2)]g_3^{K_AK\pi\pi}$$

$$+2(q.k\ P_{+\mu}-q.P_{+}k_{\mu})g_6^{K_AK\pi\pi}\ .$$

$$F_{\mu}^{K_A,S}\ (k,\ p_a,\ p_b) = \frac{G(S_{\eta}\pi\pi)}{P_+^2+M_{S_\eta}^2}\ [k_{\mu}g_1^{K_AKS_\eta}+P_{+\mu}g_2^{K_AKS_\eta}$$

$$+2(q.P_+k_{\mu}-q.kP_{+\mu})g_3^{K_AKS_\eta}+(k.qg_1^{K_AKS_\eta}+P_+.qg_2^{K_AKS_\eta})q_{\mu}/M_{K_A}^2]\ ,$$

$$G(S_{\eta}\pi\pi) = (-M_{\pi}^2+p_a.p_b)g_1^{S_\eta\pi\pi}-p_a.p_bg_3^{S_\eta\pi\pi}+g_4^{S_\eta\pi\pi}\ ,$$

$$F_{\mu}^{K_A,K^*}\ (k,\ p_a,\ p_b) = \frac{-1}{Q^2+M_{K_A}^2}\ [((g_6p_{a\mu}+g_7k_{\mu})$$

$$+g_8(Q_{\mu}+q_{\mu}Q.q/M_{K_A}^2)/M_{K^*}^2+ g_{10}\ q_{\mu}/M_{K_A}^2)g_1^{K_AK^*\pi}$$

$$-2\{(g_6p_{a\mu}+g_7k_{\mu})Q.p_b + g_{10}Q.p_bq_{\mu}/M_{K_A}^2$$

$$-g_9(Q_{\mu}-q_{\mu}Q.q/M_{K_A}^2)\}\ g_2^{K_AK^*\pi}\ -2\{(g_6p_{a\mu}+g_7k_{\mu})q.p_b$$

$$+g_8q.Qp_{b\mu}/M_{K^*}^2 + g_{10}(p_{2\mu}-Q_{\mu}q.p_b/M_{K_A}^2)\}g_3^{K_AK^*\pi}$$

$$+2\{(g_6p_{a\mu}+g_7k_{\mu})Q.q - (g_6p_a\cdot q+g_7k.q)Q_{\mu}\}g_4^{K_AK^*\pi}\ ]\ ,$$

$$Q = k - p_a\ ,$$

$$g_6 = -g_1^{K^*K\pi} - 2Q.kg_3^{K^*K\pi}\ ,$$

$$g_7 = -g_2^{K^*K\pi} + 2Q.p_ag_3^{K^*K\pi}\ ,$$

$$g_8 = g_6p_a\cdot Q + g_7\ k.Q\ ,$$

$$g_9 = g_6 p_a \cdot p_b + g_7 k \cdot p_b \quad ,$$

$$g_{10} = g_6 p_a \cdot Q + g_7 k \cdot q \quad ,$$

$$F_\mu^{K_A, S_K} = (k,\ p_a,\ p_b) = \frac{1}{Q^2 + M_{S_K}^2}\ G(S_K K\pi)\ [p_{b\mu} g_1^{K_A \pi S_K} + Q_\mu g_2^{K_A \pi S_K}$$

$$+ 2 g_3^{K_A \pi S_K}\ (q \cdot p_b Q_\mu - Q \cdot q p_{b\mu})$$

$$+ (p_b \cdot q g_1^{K_A \pi S_K} + Q \cdot q g_2^{K_A \pi S_K})\ q_\mu / M_{K_A}^2\ ] \quad ,$$

$$G(S_K K\pi) = [(M_\pi^2 + p_a \cdot k) g_1^{S_K K\pi} + (M_K^2 + p_a \cdot k) g_2^{S_K K\pi} - p_a \cdot k g_3^{S_K K\pi} - g_4^{S_K K\pi}\ ] \quad ,$$

$$F^K(k,\ p_a,\ p_b) = g_1^{K\pi} - P_+^2 g_2^{K\pi}/2 - p_a \cdot p_b g_3^{K\pi} + q \cdot k g_4^{K\pi}$$

$$+ (2 p_a \cdot p_b k \cdot q - p_a \cdot q p_b \cdot k - p_b \cdot q p_a \cdot k)\ g_7^{K\pi} \quad ,$$

$$F^{K, S_K}(k,\ p_a,\ p_b) = \frac{G(S_K K\pi)}{Q^2 + M_{S_K}^2}\ [\ -M_\pi^2 g_1^{S_K K\pi} + q^2 g_2^{S_K K\pi}$$

$$- p_2 \cdot q (g_3^{S_K K\pi} - g_1^{S_K K\pi} - g_2^{S_K K\pi}) + g_4^{S_K K\pi}\ ] \quad ,$$

$$F^{K,\ S}(k,\ p_a,\ p_b) = \frac{G(S_\eta \pi\pi)}{P_+^2 + \ _S^2}\ [(q^2 - M_K^2 - k \cdot q) g_1^{S_\eta KK} + k \cdot q g_3^{S_\eta KK}$$

$$+ g_4^{S_\eta KK}] + S_\eta \leftrightarrow S_{\eta'} \quad ,$$

$$F^{K,K^*}(k,\ p_a,\ p_b) = \frac{1}{Q^2 + M_{K^*}^2}\ [g_6 g_6' p_a \cdot p_b + g_6 g_7' p_a \cdot q$$

$$+ g_6' g_7 k \cdot p_b + g_7 g_7' k \cdot q$$

$$+ \frac{1}{M_{K^*}^2}\ (g_6 p_a \cdot Q + g_7 Q \cdot k)(g_6' p_b \cdot Q + g_7' Q \cdot q)] \quad ,$$

$$g_6' = -g_1^{K^* K\pi} - 2 Q \cdot k g_3^{K^* K\pi} \quad ,$$

$$g_7' = g_2^{K^* K\pi} + 2 Q \cdot p_b g_3^{K^* K\pi} \quad ,$$

$$F_\mu^{'K_A}(k,\ p_a,\ p_b) = 2[k.p_a p_{b\mu} - k.p_b p_{a\mu} + (k.p_a p_b.q - k.p_b p_a.q)\, q_\mu/M_{K_A}^2\,]\ g_4^{K_A K\pi\pi} + 2[P_-.qk_\mu - q.kP_{-\mu}]\ g_8^{K_A K\pi\pi} + 2[q.p_b p_{a\mu} - q.p_a p_{b\mu}] g_9^{K_A K\pi\pi} - [P_{-\mu} + P_-.qq_\mu/M_{K_A}^2\,] g_{10}^{K_A K\pi\pi} \quad ,$$

$$F^{'K}(k,\ p_a,\ p_b) = -(k+q).P_- g_5^{K\pi} + 2(q.p_b k.p_a - q.p_a k.p_b) g_6^{K\pi} \quad ,$$

$$F_\mu^{K_A,\rho}(k,\ p_a,\ p_b) = -\frac{-G(\rho\pi\pi)}{P_+^2 + M_\rho^2}\ [(P_{-\mu} + P_-.qq_\mu/M_{K_A}^2) g_1^{K_A\rho K} + 2(P_+.kP_{-\mu} - P_-.kP_{+\mu} + (P_+.kP_-.q - P_-.kP_+.q)q_\mu/M_{K_A}^2)\ g_2^{K_A\rho K} + 2(q.kP_{-\mu} - P_-.qk_\mu) g_3^{K_A K} + 2(q.P_- P_{+\mu} - q.P_+ P_{-\mu}) g_4^{K_A\rho K}\ ] \ ,$$

$$F^{K,\rho}(k,\ p_a,\ p_b) = -\frac{G(\rho\pi\pi)}{P_+^2 + M_\rho^2}\ (g_1^{\rho KK} + P_+^2\ g_3^{\rho KK})\ P_-.(q + k)$$

$$G(\rho\pi\pi) = g_1^{\rho\pi\pi} + P_+^2\ g_3^{\rho\pi\pi} \ .$$

The K_{14} decay width is given by,[45]

$$\Gamma(K \to \pi\pi l\nu) = \frac{G^2}{64(2\pi)^5 M_K^2}\int_{4M_\pi^2}^{(M_K-M_l)^2} dM_{\pi\pi}^2 \int_{M_l^2}^{(M_K-M_{\pi\pi})^2} dM_{l\nu}^2 \left[\ H_1|F_1|^2 + (H_2+H_3/3+H_4/3)|F_2|^2 + H_5|F_3|^2 + 2H_6|F_1F_3|\ \right] \ . \quad (5.73)$$

The quantities appearing in the above expression are,

$$M_{\pi\pi}^2 = -(p_a + p_b)^2 \ ,$$

$$M_{l\nu}^2 = -(p_l + p_\nu)^2 \ ,$$

$$H_1 = 2J_1J_2J_3^2\ [M_l^2\ M_{\pi\pi}^2/M_K^2 + \tfrac{2}{3}\ J_1^2\ (1+2M_l^2/M_{l\nu}^2)] \ ,$$

$$H_2 = \tfrac{2}{3}\ J_1J_2J_3^2\ M_{\pi\pi}^2\ M_{l\nu}^2\ (1+2M_l^2/M_{l\nu}^2)/M_K^2 \ ,$$

Table VI: Decay Widths (in unit of 10^3 sec^{-1}) of $K \to \pi\pi l\nu$ for Some Values of x and δ.

	x \ δ	0	0.4	0.8	1.2	Experimental Value
	1.0	2.22	2.19	2.16	2.14	
$\Gamma(K^+ \to \pi^+\pi^- e^+\nu)$	1.1	2.34	2.32	2.29	2.27	2.6 ± 0.3[a]
	1.2	2.49	2.46	2.44	2.41	
	1.0	0.93	0.92	0.91	0.90	
$\Gamma(K^+ \to \pi^0\pi^0 e^+\nu)$	1.1	1.00	0.99	0.98	0.97	
	1.1	1.07	1.06	1.05	1.04	
$\Gamma(K^0 \to \pi^-\pi^0 e^+\nu)$		0.70	0.69	0.68	0.67	
	1.0	1.19	1.20	1.20	1.20	
$\Gamma(K^+ \to \pi^+\pi^-\mu^+\nu)$	1.1	1.19	1.19	1.19	1.19	1.1 ± 0.7
	1.2	1.19	1.19	1.19	1.19	
	1.0	0.58	0.58	0.58	0.58	
$\Gamma(K^+ \to \pi^0\pi^0\mu^+\nu)$	1.1	0.58	0.58	0.58	0.58	
	1.2	0.58	0.58	0.58	0.58	
$\Gamma(K^0 \to \pi^-\pi^0\mu^+\nu)$		0.054	0.054	0.054	0.054	

a. Ref. 88.

$$H_3 = \frac{2}{3} J_1 J_2^3 J_3^3 M_{\pi\pi}^2 M_{1\nu}^2 / M_K^2 + \frac{4}{3} J_1^3 J_2^3 J_3^2 (1 + 2M_1^2/M_{1\nu}^2) ,$$
$$H_4 = \frac{4}{3} J_1 J_2^3 J_3^3 M_{\pi\pi}^2 M_{1\nu}^2 / M_K^2 ,$$
$$H_5 = 2 J_1 J_2 J_3^2 M_1^2 M_{1\nu}^2 / M_K^2 ,$$
$$H_6 = 2 J_1 J_2 J_3^2 M_1^2 (M_K^2 - M_{\pi\pi}^2 - M_{1\nu}^2)/M_K^2 ,$$
$$J_1 = [(M_K^2 + M_{\pi\pi}^2 - M_{1\nu}^2)^2 - 4M_K^2 M_{\pi\pi}^2]/4M_K^2 ,$$
$$J_2 = 1 - 4M_\pi^2/M_{\pi\pi}^2 ,$$
$$J_3 = 1 - M_1^2/M_{1\nu}^2 ,$$

In the calculations of the K_{e4} width the F_3 terms are dropped because their coefficients are proportional to m_e^2. However, $K_{\mu 4}$ widths are calculated with full expression (73). The width $\Gamma(K^0 \to \pi^-\pi^0 l^+\nu)$ is function of δ only whereas $\Gamma(K^+ \to \pi^0\pi^0 l^+\nu)$ is function of both δ and x. The decay widths of $K \to \pi\pi l\nu$ for some values of x and δ are given in the Table VI. We see that variation with δ is small, variation with x is also small in contrast to π-π scattering lengths. Upto date only $K^+ \to \pi^+\pi^- e^+\nu$ decay has been observed, some events of $K^+ \to \pi^+\pi^-\mu^+\nu$ are also reported. The experimental decay widths are[88],

$$\Gamma(K^+ \to \pi^+\pi^- e^+\nu) = (2.6 \pm 0.3) \times 10^3 \text{ sec}^{-1} , \quad (5.74)$$

and $$\Gamma(K^+ \to \pi^+\pi^-\mu^+\nu) = (1.1 \pm 0.7) \times 10^3 \text{ sec}^{-1} . \quad (5.75)$$

Our calculated values are in agreement with the experimental values. Another experimentally observed quantity is the dipion

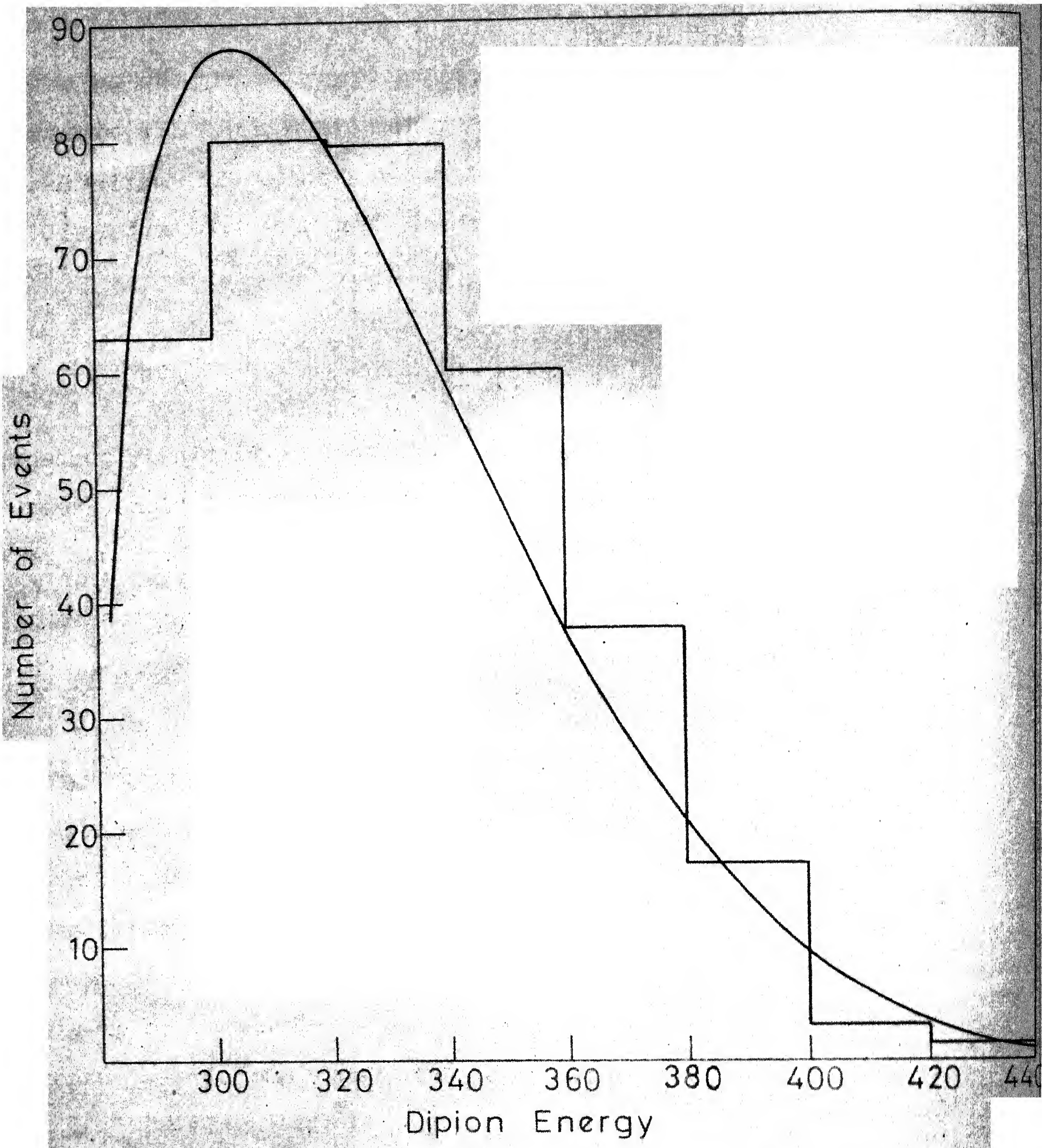

Fig. 4 Dipion energy spectrum of $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$

mass distribution i.e. $d\Gamma/dM^2_{\pi\pi}$ vs. $M_{\pi\pi}$. In Fig. 4, we have plotted $d\Gamma/dM^2_{\pi\pi}$ vs. $M_{\pi\pi}$ for $x = 1.2$ and $\delta = 0$. The experimental histogram[88] is also plotted for comparison. The variation of form factors in the physical region is for $x = 1.2$ and $\delta = 0$,

$$\begin{aligned} F_1^+ &= 3.23 \sim 3.77 \ , \\ F_2^- &= -3.20 \sim -3.67 \ , \\ F_3^+ &= -27.9 \sim -31.5 \ . \end{aligned} \tag{5.76}$$

Thus form factors can be taken as constant within 15 percent. This is in contrast to most of the other calculations where these are assumed to be constant throughout the physical region.

The K_{l4} decay form factors and the decay widths have been calculated by many authors[89]. In the following, we give a brief summary of the recent works.

The current algebra techniques with soft pions[90] give $F_1 = F_2 = 3.7$ at the unphysical point $p_a.k = p_b.k = p_a.p_b = 0$. The decay width* $\Gamma(K^+_{e4})$ implied by these form factors is 1.88×10^3 sec^{-1}. Chhajlany, Pandit and Rajasekaran[91] have recalculated these form factors with the same assumptions. They take $F_K/F_\pi = 1$, $\sin\theta = 0.22$ but assume breakdown of $\mu - e$ universality in strangeness changing current. These authors obtain $F_1 = F_2 = 5.3$, $\Gamma(K^+_{e4}) = 2.6 \times 10^3$ sec^{-1} and $\Gamma(K^+_{\mu 4}) = 0.31 \times 10^3$ sec^{-1}. The form factors are still at an unphysical

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* In the rest of the section $\Gamma(K^+_{e4})$ stand for $\Gamma(K^+ \to \pi^+\pi^- e^+\nu)$ and $\Gamma(K^+_{\mu 4})$ for $\Gamma(K^+ \to \pi^+\pi^-\mu^+\nu)$.

point. These form factors are different from those of Weinberg because Weinberg took $f_+(q^2 = -M_K^2) = 1$ whereas these authors determined it from the K^* pole dominant model.

Greenberg[92] used current algebra techniques and calculated K_{14} form factors at mass shell. He further made use of Weinberg sum rules, and approximated current matrix elements by K^*, K_A, K and ρ poles. Then, with $F_K/F_\pi = 1$ and $\sin\theta_A = 0.265$ he gets $F_1 = 4.0$, $F_2 = 4.6$, $F_3 = 1.06$ and $\Gamma(K^+_{e4}) = 2.3 \times 10^3\ \text{sec}^{-1}$.

Biswas, Dutt and Gupta[93] wrote down once substracted dispersion relations for $F_{1,2,3}$. The subtraction constants were determined from current commutators. The sum rules were saturated by ρ and K^* states. The inputs $F_K/F_\pi = 1.17$ and $\sin\theta_A = 0.265$ yield $F_1 = 3.8$, $F_2 = 4.6$ and $F_3 = 4.3$ at $k.p_a = k.p_6 = p_a.p_b = 0$, $\Gamma(K^+_{e4}) = 2.06 \times 10^3\ \text{sec}^{-1}$ and $\Gamma(K^+_{\mu 4}) = 0.33 \times 10^3\ \text{sec}^{-1}$.

The ward identity approach is used in ref. (94-95). Dutt, Gupta and Vaishya[94] wrote down three and four point functions. As usual they approximate appropriate current matrix elements by K^*, ρ, K and K_A poles and work in the limit $p_a^2, p_b^2 \to 0$ (semi soft meson approach). With inputs $F_K/F_\pi = 1.17$ and $\sin\theta_A = 0.26$ they get $F_1 = 5.85$, $F_2 = 9.4$, $F_3 = 8.95$ at the point $k.p_a = k.p_b = p_a.p_b = 0.32M_K^2$ and $\Gamma(K^+_{e4}) = 2.83\text{x}10^3\text{sec}^{-1}$. They also note large variation of form factors in the physical region. Sarker[95] also wrote down three and four point functions but kept mesons on mass shell. He obtained relation between K_{13} and K_{14} form factors. In addition of $F_K/F_\pi = 1.28$ and

$\sin\theta_A = 0.22$ he took $\xi_{K13} = 0.6\ (-1)$. This gives $F_1 = 4.8$, $F_2 = 2.66$, $F_3 = 4.4$ and $\Gamma(K^+_{e4}) = 2.1 \times 10^3\ \sec^{-1}$ for $\xi_{K13} = 0.6$ whereas $\xi_{K13} = -1$ gives $\Gamma(K^+_{e4}) = 0.92 \times 10^3\ \sec^{-1}$. These authors use full momentum dependent form factors in calculating life time.

Biswas et al[96] use nonlinear SU(3) X SU(3) invariant effective Lagrangian with spin one mesons as gauge fields. They obtain $F_1 = 4.3$, $F_2 = 3.05$, $F_3 = 2.6$ at $k.p_a = k.p_b = p_a.p_b = 0$. The inputs $\sin\theta_A = 0.22$ and $F_K/F_\pi = 1$ give $\Gamma(K^+_{e4}) = 2.43 \times 10^3\ \sec^{-1}$ and $\Gamma(K^+_{\mu 4}) = 0.31 \times 10^3\ \sec^{-1}$

Following points in connection with these works are in order. (i) Almost all the calculations use F_K/F_π and $\sin\theta_A$ as input. From (21) we note that if $F_K/F_\pi \sim 1.2$ then $\sin\theta_A$ 0.22 and if $F_K/F_\pi \sim 1$ then $\sin\theta_A \sim 0.26$. It is interesting that most of the authors have taken F_K/F_π and $\sin\theta_A$ in wrong combinations. Therefore correct combination will alter their results considerably. (ii) Most of the authors use Weinberg sum rules. Although the first sum rule is valid in most of the cases, the second sum rule is not[97] (this is the case in our model also). (iii) None of these authors take account of S_K, S_η and $S_{\eta'}$ intermediate states into consideration. Although large scalar mass in the denominator suppresses the matrix element but the large couplings, which are reflected from the large widths of S_K and S_η, compensate this. Therefore one would expect large contributions from the S_K and S_η intermediate graphs. This

is indeed the case in the present model. Our calculations should be more reliable because we do not have any of above mentioned draw backs.

We have mentioned earlier that vector current contribution to K_{14} decay is zero. This is due to the absence of VVP couplings. It has been shown[16] that so long as the Lagrangian gives partial conservation of currents and spin one fields in the Lagrangian occur only through covariant derivatives, VVP couplings do not appear. This is so because invariants involving VVP couplings satisfying these requirements have opposite C - parity compared to invariants already present. Recently some authors have introduced VPP couplings as the explicit symmetry breakihg terms and calculated vector contribution. In order to obtain estimates of vector form factors some authors have adopted the following attitude[94]. They calculate total decay width with axial vector form factors. Since it is usually less than the experimental number they demand that rest of width is due to vector form factors. This estimate is obviously highly model dependent.

CHAPTER VI

CONCLUDING REMARKS

In this chapter we discuss, after some general comments on the model, what modifications will occur if ninth vector and axial vector mesons are included. Then we analyse the consequences of the situation when explicit symmetry breaking terms are absent. Finally a limit of the model is considered in which masses of chiral partners of the Goldstones become infinite.

6-1 General Comments:

In this work we have employed the effective Lagrangian method to study low energy processes of spin zero and spin one mesons. The predictions of the model are in reasonably good agreement with the available experimental data. Further some of our results like $\rho\pi\pi$, $A_1\rho\pi$ vertex and K_{l3} form factors agree with those obtained from pole dominance and smoothness assumption of two, three and four point functions. It is expected because in tree graph approximation these assumptions are built-in in Lagrangian approach; smoothness assumption comes from the fact that in invariants maximum number of $F_{\mu\nu}$ or $G_{\mu\nu}$ is two. Wherever difference arises, it is due to those assumptions which are not valid in our model, for example, conservation of strangeness changing vector current, Weinberg second sum rule and absence of scalar particles.

Our model gives a good description of particle masses. The predicted masses, except K^* mass which is within 5 percent of experimental mass, are close to their experimental values. We are able to predict scalar masses which are within 3 percent of the corresponding experimentally reported peaks. We consider it, one of the successes of the model.

The overall agreement between predicted and experimental decay widths is good. However, width of two particle decay mode of K_A is rather large.

An important feature of the model is that it makes definite predictions about scalar mesons contributions to various processes. In most of the cases like $\pi - \pi$, $K - \pi$ and $K - K$ scattering and K_{l4} decay, scalar meson contributions are significant. Moreover decay widths of those modes in which scalar mesons occur either as decaying particle or in final state are not negligible. The future measurements of these processes will constitute a test of the model.

Apart from the usual shortcomings of the Lagrangian approach, like unitarity, one of the shortcoming is that we have only octets and not nonets of vector and axial vector mesons. This point is discussed in the following subsection.

6-2 Inclusion of Ninth Vector and Axial Vector Mesons:

Throughout this investigation, we had octets of vector and axial vector mesons. However, experimentally there are two $I = 0$, $Y = 0$ vector mesons, ω (783) and ϕ (1019), and two axial vector mesons, D (1286) and E (1420). In order to include all of these in the model, we must start with two $I = 0$, $Y = 0$ vector mesons and two $I = 0$, $Y = 0$ axial vector mesons. The ninth vector and axial vector mesons can be included in the Lagrangian by assuming that they belong to (1, 1) + (1, 1) representation of SU(3) X SU(3). The physical $I = 0$, $Y = 0$ spin one mesons would then appear as the mixed states of the two $I = 0$, $Y = 0$ members of the multiplet. In the nonet theory Y_μ and Z_μ (2.47) would represent nonets (instead of octets). By nonet

theory we mean that in the absence of symmetry breaking all the nine vector (axial) mesons are degenerate and only one coupling constant in the covariant derivatives occurs. The ninth fields, however, do not appear as gauge fields because the regular representation of SU(3) X SU(3) is only sixteen dimensional. If the symmetry is extended to the larger group U(3) X U(3), which is eighteen parameter group, then all the spin one mesons would appear as gauge fields.[98]

In the SU(3) X SU(3) model with nonets of spin one mesons, the mixing between eight and ninth spin one meson comes from the invariants corresponding to h_1 and h_2. When this mixing is removed one finds that ninth member becomes degenerate with I = 1, Y = 0 member of the octet[16]. In the vector meson case, this equality is satisfied well, by $\omega(783)$ and $\rho(765)$, but in the axial vector case it means that A_9 is degenerate with $A_1(1070)$, far from the possible candidate D(1286). Further ψ, the kinetic energy mixing angle of eight and ninth pseudoscalar (3.10), becomes 35^0 (mixing angle in corresponding vector and axial vector mesons is also equal to 35^0)[99]. This can be avoided if we include invariants like $\{D_\mu M D_\mu M^+ M M^+\}$ and $\{D_\mu M M^+ D_\mu M M^+\}$ which contribute to the renormalizations of spin zero mesons.

In order to break $\rho - \omega$ and $A_1 - A_9$ degeneracy more invariants like $\{F_{\mu\nu}\}^2$ and $\{G_{\mu\nu}\}^2$ have to be included.[100] When this is done, we find that, it is not possible to fit D(1286) and E (1420). For almost all the ranges of parameters which

keep other masses close to experimental values, A_9 continues to be around 1000 MeV and A_8 around 1500 MeV or greater. This difficulty may be overcome if one further includes the singlet mass term for the ninth spin one fields i.e. invariants like $\{V_\mu\}^2$ and $\{A_\mu\}^2$. Now eight spin one masses are function of nine parameters, therefore, it is very difficult to fit the parameters. Further with so many parameters the model looses much of its predictive power. Thus we see that by leaving out ninth spin one mesons, the model is greatly simplified and still gives good agreement with the experimental data.

6-3 Purely Spontaneous Symmetry Breaking:

Let us consider the situation when the explicit symmetry breaking is absent i.e. $b_8 = b_o = 0$. For convenience in the following, we again write down (3.16) and (3.17),

$$f_o\mu_o^2 + \frac{3\alpha}{\sqrt{2}}(f_o^2 - f_8^2/3) + \beta(f_o^3 + 2f_o f_8^2 - 2f_8^3/3\sqrt{3}) = \sqrt{2/3}\, b_o \,, \qquad (6.1)$$

$$f_8\,(\mu_o^2 - \frac{3\alpha}{\sqrt{2}}(f_o + f_8/\sqrt{3}) + 3\beta(f_o^2 - f_o f_8/\sqrt{3} + f_8^2/3)) = b_8 \,. \qquad (6.2)$$

Let us first consider the case when only $b_8 = 0$, i.e. Lagrangian is SU(3) symmetric. Eqs. (2), (3.28), (3.29) and (5.9) give,

$$b_8 = \frac{2}{\sqrt{3}}\, M_{S_K}^2\, F_{S_K}\, Z_{S_K}^{-1/2} \,. \qquad (6.3)$$

With $b_8 = 0$, (3) implies that at least one of the three quantities on the right hand side must vanish. The case $M_{S_K}^2 = 0$ correspond to the prediction of the Goldstone theorem in the case of a purely spontaneous breakdown of SU(3) to the isospin-

hypercharge level. If $F_{S_K} = 0$ and $|Z_{S_K}| < \infty$, (5.12) implies $f_8 = 0$, which corresponds to the situation when SU(3) symmetry remains unbroken by the vacuum as well. Finally from the relation (3.29),

$$Z_{S_K} = 1 + 3g^2f_8^2/4m_o^2 \quad , \tag{6.4}$$

we see that the case $|Z_{S_K}| = \infty$ correspond to $m_o = 0$ (assuming that gf_8 is nonzero and finite). In this case there will be no kinetic energy term for the S_K meson and this particle is eliminated from the theory. This is what one expects from the work of Higgs[11] and Kibble[12]. These authors have shown that when symmetry is broken spontaneously in a fully gauge invariant theory, the Goldstones become the longitudinal modes of the corresponding gauge fields and are eliminated from the theory.

Now let us consider the case when $b_8 = b_o = 0$ i.e. Lagrangian is SU(3) X SU(3) symmetric. In this case, we have the following situations:

(1) One can see that $f_o = f_8 = 0$ is one of the solution of (1) and (2) with $b_o = b_8 = 0$ so that vacuum is SU(3) X SU(3) symmetric.

(2) $f_8 = 0$ and $f_0 \neq 0$. In this case, (1) gives $\mu_o^2 + 3\alpha f_o/\sqrt{2} + \beta f_o^2 = 0$. The symmetry of vacuum is SU(3). Eq. (3.23) gives $M_\pi = M_K = M_\eta = 0$ ($M_{\eta'}$ is nonzero and finite), i.e. octet of pseudoscalar appears as Goldstones.

(3) $f_0 \neq 0$, $f_8 \neq 0$. The elimination of μ_0^2 from (1) and (2) with $b_0 = b_8 = 0$ gives,

$$(2f_0 - f_8/\sqrt{3})(f_0 + f_8/\sqrt{3})\ (3\alpha/\sqrt{2} - \beta(f_0 - 2f_8/\sqrt{3})) = 0. \quad (6.5)$$

We have three situations depending on which factor in (5) is zero (i). $f_0 + f_8/\sqrt{3} = 0$. The symmetry of the vacuum is SU(2) X SU(2) X U(1). In this case, S_K, K and η mesons appear as Goldstones (ii) $2f_0 - f_8/\sqrt{3} = 0$. The pions, S_K and -mesons are Goldstones. The symmetry is an unusual SU(3) corresponding to $Q^V_{1,2,3,8}$ and $Q^A_{4,5,6,7}$. (iii) $3\alpha/\sqrt{2}$ - $-\beta(f_0 - 2f_8/\sqrt{3}) = 0$. The symmetry of the vacuum is SU(2) X U(1). One can see from (3.23) that π, K, η and S_K masses are zero, therefore, these appears as Goldstones.

6-4 A Limit of the Model:

It is of some interest to consider a limit of the model in which the chiral partners of the Goldstones, that is η', S_π, S_η and $S_{\eta'}$, are eliminated from the model by letting their masses go to infinity. Similar limit in SU(2) X SU(2) σ model was considered by Weinberg i.e. $M^2_\sigma \to \infty$ which yielded current algebra results for N - π and π - π scattering lengths. In this limit, we hope to establish correspondance with results obtained from current algebra and nonlinear Lagrangians.

The limit $M_{\eta'}$, M_{S_π}, M_{S_η} and $M_{S_{\eta'}}$ $\to \infty$ is implemented by letting μ_0^2, α, β, γ $\to \infty$ such that ratio of any two of them is finite. In this limit only those quantities will be effected

which depend on μ_0^2, α, β and γ. In order to obtain various quantities in this limit the following procedure is adopted. First, the quantities which depend on μ_0^2, α, β and γ are expressed as function of β and γ with the help of (1) and (2). Then the limit β, $\gamma \to \infty$ is taken. It so happens that the quantities which stay nonzero and finite in this limit are independent of the ratio β/γ. Following are the results in this limit. (Henceforth the model discussed so far will be referred as model A and the model in the above mentioned limit as model B.).

(1) The spin one masses are not effected. The masses of Goldstones are nonzero and finite. The expressions for the masses are,

$$M_\pi^2 = Z_\pi \left(\sqrt{2/3}\, b_0 + b_8/\sqrt{3}\right)/\left(f_0 + f_8/\sqrt{3}\right), \tag{6.7}$$

$$M_K^2 = Z_K \left(\sqrt{2/3}\, b_0 - b_8/2\sqrt{3}\right)/\left(f_0 - f_8/2\sqrt{3}\right), \tag{6.8}$$

$$M_\eta^2 = Z_\eta\left(\left(f_0 - f_8/\sqrt{3}\right)\sqrt{2/3}\, b_0 - \left(f_0 - 3 f_8/\sqrt{3}\right) b_8/\sqrt{3}\right) \Big/ \left(f_0^2 - 2 f_0 f_8/\sqrt{3} + f_8^2/3\right). \tag{6.9}$$

$$M_{S_K}^2 = Z_{S_K}\; b_8/f_8 . \tag{6.10}$$

In fact (7), (8) and (10) are same as those obtained from (5.38) i.e. the expressions for π, K and S_K masses in the two models A and B are same. However, the expression for η mass is different in two models. In model A the η and η' masses are given by (3.27). In model B one finds that $(M_p)^2_{88}$ and $(M_p)^2_{80}$,

defined by (3.23), are unaffected by the limit whereas $(M_p^2)_{oo}$ becomes infinite. Therefore θ_p (3.26) becomes zero and $M_\eta^2 \rightarrow (M_p)_{88}^2$.

(2) The currents remain unchanged. But since $\theta_p = 0$, $F_{\eta'}$ (see 5.11) becomes zero. The divergence of the eight axial vector current (5.38) now becomes,

$$\partial_\mu J^A_{\mu 8} = F_\eta M_\eta^2 \eta + \sqrt{\tfrac{2}{3}} \frac{(\sqrt{2/3}\,b_o f_8 - b_8 f_o)}{(f_o^2 - 2 f_o f_8/\sqrt{3} + f_8^2)^{1/2}} \eta' . \quad (6.11)$$

The nonzero coefficient of η' appears because $M_{\eta'}^2 F_{\eta'}$ is finite although $M_{\eta'}^2 \rightarrow \infty$ and $F_{\eta'} \rightarrow 0$. In the pole dominance model of divergence of current the η' state, due to nonzero coefficient, will also contribute to the matrix elements of (11). However, since $M_{\eta'}^2 \rightarrow \infty$, its contribution will be zero.

The K_{13} form factors in model B are same as in model A. This is so because S_K couplings and mass are not affected by the limit.

(3) The decay widths of η', S_π, S and $S_{\eta'}$ become infinite. This is expected because their masses themselves go to infinity. Those decay modes in which any of η', S_π, S_η and $S_{\eta'}$ appear in final state are not allowed. The amplitude of the processes like three particle decay modes, scattering and K_{14}, where these particles appear as intermediate state are changed. The η', S_π, S_η and $S_{\eta'}$ pole diagrams become contact terms which are nonzero and finite.

The π-π scattering amplitude A(s, t, u) (4.72) in this model is given by,

$$A(s,\ t,\ u) = \left[(g_1^{\rho\pi\pi} + u g_3^{\rho\pi\pi})^2\ (s-t)/(M^2-u) + u \leftrightarrow t\right]$$
$$+\ 2g_3^{\pi\pi}\left[u^2+t^2-2s^2-4M_\pi^2(u+t-2s)\right]$$
$$+\ g^2\xi_\pi^2\ Z_\pi^2\ (s - 2M_\pi^2) + g^2 Z_\pi(4M_\pi^2-3s)/m_o^2$$
$$+\ (s-M_\pi^2)/F_\pi^2\ . \tag{6.12}$$

The π-π amplitude obtained from current algebra and nonlinear Lagrangian approach[3] result has only the last term of (12). It is interesting to note that in the limit $g \to 0$ all the remaining terms except last term in (12) vanish. This is in fact expected because model B in the limit $g \to 0$ is the same as the one considered by Weinberg. The K-π amplitude even in the limit $g \to 0$ differs from current algebra result by the contribution of S_K pole which is usually neglected in these calculations.[7]

The s-wave π-π scattering lengths as given by (12) are,

$$32\pi a_{00} = M_\pi\left[16(g_1^{\rho\pi\pi})^2/M_\rho^2 + 7/F_\pi^2+2g^2\xi_\pi^2 Z_\pi^2-16g^2 Z_\pi/m_o^2\right],$$
$$32\pi a_{02} = M_\pi\left[-8(g_1^{\rho\pi\pi})^2/M_\rho^2 - 2/F_\pi^2-4g^2\xi_\pi^2 Z_\pi^2+ 8g^2 Z_\pi/m_o^2\right]. \tag{6.13}$$

It is interesting to see what happens when KSRF is assumed. The KSRF is obtained if the following condition is satisfied,

$$m_o^2 = g^2(f_o + f_8/\sqrt{3})^2\ . \tag{6.14}$$

In this limit $Z_\pi = 2$, (3.24), $\xi_\pi^2 = 1/4m_o^2$, $F_\pi^2 = m_o^2/2g$ (5.11) and $(g_1^{\rho\pi\pi})^2/M_\rho^2 = g^2/m_o^2$. With the help of these it is easy to see

that $a_{00} = a_{02} = 0$.

(4) In model B total number of parameters are nine, two less than the model A. Since the two models are different from each other one should determine all the parameters again. The procedure of determining parameters from spin one masses remain same as in model A. To determine b_0 and b_8 we take π and K masses as input. The ξ is restricted such that S_K mass is between 1080 and 1200 MeV, corresponding to the mass of experimentally reported resonance. It means ξ is between -0.08 and and -0.095. The overall best fit is obtained for $\xi = -0.08$. The predicted particle masses are,

$$M_{K^*} = 935 \text{ MeV}, \quad M_{K_A} = 1308 \text{ MeV}$$
$$M_{\eta} = 576 \text{ MeV}, \quad M_{S_K} = 1186 \text{ MeV} .$$

The widths (in MeV) of two particle decay modes of vector and axial vector mesons and three particle decay modes of axial vector mesons for $\delta = 0.6$ are,

$$\Gamma(\rho \to \pi\pi) = 117$$
$$\Gamma(K^* \to K\pi) = 55.5$$
$$\Gamma(\rho \to \bar{K}K) = 3.7$$
$$\Gamma(A_1 \to \rho\pi) = 135$$
$$\Gamma(K_A \to \rho K) = 9.3$$
$$\Gamma(K_A \to K^*\pi) = 255$$
$$\Gamma(E \to \bar{K}^*K) = 27.4$$
$$\Gamma(A_1 \to 3\pi) = 140$$

$$\Gamma(K_A \to K\pi\pi) = 121$$
$$\Gamma(E \to \bar{K}K\pi) = 12$$
$$\Gamma(E \to \eta\pi\pi) = 37$$

The s-wave I = 0, I = 2 and p-wave I = 1 scattering lengths and effective ranges, for $\pi\pi$, for $\delta = 0$ are,

$$a_{00} = 0.117\, M_\pi^{-1} \quad , \quad r_{00} = -55\ M_\pi^{-1} \quad ,$$
$$a_{02} = -0.042\, M_\pi^{-1} \quad , \quad r_{02} = -11\ M_\pi^{-1} \quad ,$$
$$a_{11} = 0.032\, M_\pi^{-1} \quad , \quad r_{11} = 65\ M_\pi^{-1} \quad .$$

The s-wave I = 3/2 and I = 1/2 K-π scattering lengths and effective ranges are,

$$a_0^{3/2} = 0.63\, M_\pi^{-1} \quad , \quad r_0^{3/2} = 0.20\, M_\pi^{-1} \quad ,$$
$$a_0^{1/2} = 0.47\, M_\pi^{-1} \quad , \quad r_0^{1/2} = 1.85\, M_\pi^{-1} \quad .$$

The s-wave low energy parameters for K-K scattering are,

$$a_0^1\ (KK) = 0.39\, M_\pi^{-1} \ , \quad r_0^1 = 1.3\ M_\pi^{-1}$$

The π-π scattering lengths predicted by model B can also be obtained in model A for some suitable value of x (see Table IV). The ratio a_{00}/a_{02} is in agreement with experiments.[49,50] The K-π scattering lengths are large and of opposite sign as compared to the values of model A. This is so because in model A, S_K and $(S_\eta, S_{\eta'})$ contributions were of same magnitude but of opposite sign. However, in model B, S_K contribution which is positive dominates over the contact diagram contribution. The contribution of $(S_\eta, S_{\eta'})$ diagrams in model A to K-K scattering

was much large than the corresponding contribution in model B. Therefore, here K-K scattering length is small.

The K_{13} parameters for $\delta = 0.6$ are as follows,

$f_+(0) = 0.99$; $\lambda_+ = 0.020$,

$\lambda_- = 0.023$, $\xi_{K13} = -0.203$,

$\Gamma(K^+_{\mu 3})/\Gamma(K^+_{e3}) = 0.641$.

The width of $K^0 \to \pi^- \pi^0 e^+ \nu$ and F_2^- (5.76) are not effected. The widths of the mode $K^+ \to \pi^+ \pi^- l^+ \nu$ for $\delta = 0.6$ are,

$$\Gamma(K^+ \to \pi^+\pi^- e^+\nu) = 0.93 \times 10^3 \text{ sec}^{-1} ,$$

$$\Gamma(K^+ \to \pi^+\pi^- \mu^+\nu) = 1.09 \times 10^3 \text{ sec}^{-1} .$$

The variation of K_{14} form factors in the physical region is as follows,

$$F_1^+ = 1.84 \sim 1.72 ,$$

$$F_3^+ = 16 \sim 17 .$$

We see that F_1 is smaller than the corresponding value in model A (5.76). It explains small width for $K^+ \to \pi^+\pi^- e^+\nu$ mode. The reason is that the contribution of $(S_\eta, S_{\eta'})$ states to F_1 in model A is positive whereas corresponding contribution in model B is negative. The contribution of contact term to F_3 in model B is negative but about 15 times smaller than the $(S_\eta, S_{\eta'})$ contribution in model A. Since in model B, F_1 and F_3 are of same sign (in model A they are of opposite sign) therefore, contribution of interference term F_1 F_3 to width is positive. This explain why widths of $K^+ \to \pi^+\pi^- \mu^+\nu$ in two model is of same order.

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$$+ g_T\, \epsilon_{\lambda\alpha\beta\gamma}\, \epsilon_{\mu\alpha'\beta'\gamma}\, Q_\alpha q_\beta Q_{\alpha'} q_{\beta'} \epsilon^A_\lambda\, \epsilon^\rho_\mu$$

The coupling constants G_S and G_D are related to g_L and g_T through the relation,

$$G_S = g_T (M_V^2 - (Q.q)^2/M_A^2) ,$$

$$G_D = g_L + Q.q\, g_T/M_A^2 .$$

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APPENDIX

COUPLING CONSTANTS

In this appendix, we write down the effective Lagrangian for VPP, AVP, APS, SPP, APPP and PPPP vertices. Eq. (3.7) in the matrix form is parametrised as,

$$f = \begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & d \end{pmatrix} \quad , \qquad (A.1)$$

where,

$$c = \frac{1}{\sqrt{2}} (f_0 + f_8/\sqrt{3}) \quad ,$$

$$d = \frac{1}{\sqrt{2}} (f_0 - 2f_8/\sqrt{3}) \ .$$

We define two 3 x 3 matrices namely Φ_μ and Π_μ such that (3.8) in matrix notation becomes,

$$Y_\mu = \hat{Y}_\mu + \Phi_\mu \quad ,$$

$$Z_\mu = \hat{Z}_\mu + \Pi_\mu \quad , \qquad (A.2)$$

where,

$$\Phi_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i \xi^s_{ij} \partial_\mu \phi_j ,$$

$$\Pi_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i \xi^p_{ij} \partial_\mu \pi_j$$

Two more 3 x 3 matrices S_μ and P_μ are defined through the relation,

$$S_\mu = \partial_\mu \Phi + \frac{ig}{\sqrt{2}} [\Phi_\mu, f]_- \quad , \qquad (A.3)$$

and,

$$P_\mu = \partial_\mu \Pi - \frac{g}{\sqrt{2}} [\Pi_\mu , f]_+ \quad . \qquad (A.4)$$

With the help of (2), (3) and (4) the covariant derivatives $D_\mu \Phi$, $D_\mu \Pi$, $F_{\mu\nu}$ and $G_{\mu\nu}$ now can be written as,

$$D_\mu \Phi = S_\mu + \frac{ig}{\sqrt{2}} [\hat{V}_\mu , f + \Phi]_- + \frac{ig}{\sqrt{2}} [\Phi_\mu , \Phi]_- + \frac{g}{\sqrt{2}} [\hat{Z}_\mu + \Pi_\mu , \Pi]_+ \quad ,$$

$$D_\mu \Pi = P_\mu + \frac{ig}{\sqrt{2}} [\hat{V}_\mu + \Phi_\mu , \Pi]_- - \frac{g}{\sqrt{2}} [\hat{Z}_\mu , \Phi + f]_+ - \frac{g}{\sqrt{2}} [\Pi_\mu , \Phi]_+ \quad ,$$

$$F_{\mu\nu} = \hat{Y}_{\mu\nu} + \frac{ig}{\sqrt{2}} [\hat{Y}_\mu + \Phi_\mu , \hat{Y}_\nu + \Phi_\nu]_- + \frac{ig}{\sqrt{2}} [\hat{Z}_\mu + \Pi_\mu , \hat{Z}_\nu + \Pi_\nu]_- \quad .$$

$$G_{\mu\nu} = \hat{Z}_{\mu\nu} + \frac{ig}{\sqrt{2}} [\hat{Y}_\mu + \Phi_\mu , \hat{Z}_\nu + \Pi_\nu]_- + \frac{ig}{\sqrt{2}} [\hat{Y}_\nu + \Phi_\nu , \hat{Z}_\mu + \Pi_\mu]_- \, . \qquad (A.5)$$

where $\hat{Y}_{\mu\nu} = \partial_\mu \hat{Y}_\nu - \partial_\nu \hat{Y}_\mu$ etc.

The fields π_8 and π_0 are related to physical fields through the relations,

$$\begin{aligned} \pi_8 &= (Z_\eta^{1/2} \cos\psi \cos\theta_p - \sin\psi \sin\theta_p)\, p_\eta \\ &\quad + (Z_\eta^{1/2} \cos\psi \sin\theta_p + \sin\psi \cos\theta_p)\, p_{\eta'} \\ &= C_{8\eta}\, p_\eta + C_{8\eta'}\, p_{\eta'} \quad , \\ \pi_0 &= (-Z_\eta^{1/2} \sin\psi \cos\theta_p - \cos\psi \sin\theta_p)\, p_\eta \\ &\quad + (-Z_\eta^{1/2} \sin\psi \sin\theta_p + \cos\psi \cos\theta_p)\, p_{\eta'} \\ &= C_{0\eta}\, p + C_{0\eta'}\, p \, , \quad . \end{aligned} \qquad (A.6)$$

Similarly s_8 and s_0 will be replaced by,

$$s_8 = \cos\theta_s \, S_\eta + \sin\theta_s \, S_{\eta'} \,,$$

$$s_0 = -\sin\theta_s \, S_\eta + \cos\theta_s \, S_{\eta'} \,. \tag{A.7}$$

The parameter δ with which variation of most of the quantities is given,. is defined as,

$$\delta = h\, M_\rho^2 / g = \delta' \, M_\rho^2 \,. \tag{A.8}$$

The other constants which are frequently used in the following are,

$$\xi_\pi = \sqrt{2}\, gc/(m_o^2 + 2\, g^2 c^2) \,,$$

$$\xi_K = \frac{g}{\sqrt{2}} (c + d)/(m_o^2 + g^2 (c + d)^2/2) \,,$$

$$\xi_{88} = \frac{\sqrt{2}}{3} g(c + 2d)/(m_o^2 + \frac{2}{3} g^2 (c^2 + 2d^2)) \,,$$

$$\xi_{80} = \frac{2g}{3} (c - d)/(m_o^2 + \frac{2}{3} g^2 (c^2 + 2d^2)) \,,$$

$$\xi_s = \frac{g}{\sqrt{2}} (c - d)/(m_o^2 + \frac{g^2}{2} (c - d)^2),,$$

$$L_\pi = 1 - \sqrt{2}\, gc\, \xi_\pi \,, \quad L_K = 1 - g(c + d)\, \xi_K/\sqrt{2} \,,$$

$$L_{81} = 1 - \sqrt{2}\, gc\, \xi_{88} \,, \quad L_{82} = 1 - \sqrt{2} gd\, \xi_{88} \,,$$

$$L_{01} = 1 - gc\, \xi_{80} \,, \quad L_{02} = 1 + 2gd\, \xi_{80} \,,$$

$$L_S = 1 - g(c - d)\, \xi_S/\sqrt{2} \,,$$

$$h_4 = 1 + (h_1 + h_2)\, c^2 \,,$$

$$h_5 = 1 + (h_1 + h_2)(c^2 + 2d^2)/3 \,,$$

$$h_6 = 1 + \frac{1}{2} h_1 (c^2 + d^2) + h_2\, cd \,,$$

$$h_7 = 1 + (h_1 - h_2)\, c^2 \;,$$
$$h_8 = 1 + \frac{h}{2}(c^2 + d^2) - h_2\, cd \;. \tag{A.8}$$

The renormalization constants for vector fields are given by (3.20), for axial vector fields by (3.22) and for spin zero fields by (3.24) and (3.29). In the following, particle symbols stand for the corresponding fields.

(1). VPP Vertex:

$$\begin{aligned}
L(VPP) = &-\frac{ig}{\sqrt{2}}\Big\{[\hat{Y}_\mu, \Pi]_- P_\mu + \frac{g}{\sqrt{2}}[\hat{Y}_\mu, f]_+ [P_\mu, \Pi]_- \\
&+ \frac{1}{2}\hat{Y}_{\mu\nu}([\Pi_\mu, \Pi_\nu]_- - \delta'[P_\mu, P_\nu]_-)\Big\} \\
&+ \frac{h_1}{4}\Big\{\hat{Y}_{\mu\nu}([\Pi_\mu, \Pi_\nu]_- f^2 + f^2[\Pi_\mu, \Pi_\nu]_-)\Big\} \\
&+ \frac{h_2}{2}\Big\{\hat{Y}_{\mu\nu}\; f\,[\Pi_\mu, \Pi_\nu]_-\, f\Big\}\;. \qquad\qquad (A.9) \\
= \;& g_1^{\rho\pi\pi}\, \rho_\mu\cdot\pi \times \overleftrightarrow{\partial}_\mu \pi + g_3^{\rho\pi\pi}\, \rho_{\mu\nu}\, \partial_\mu\pi \times \partial_\nu\pi \\
&+ i\, g_1^{\rho KK}\, \rho_\mu\, K^+\tau\, \overleftrightarrow{\partial}_\mu K + i\, g_3^{\rho KK}\, \rho_{\mu\nu}(\partial_\mu K^+\tau\, \partial_\nu K \\
&\qquad - \partial_\nu K^+\tau\, \partial_\mu K) \\
&+ i\, g_1^{\phi KK}\, \phi_\mu K^+\cdot\overleftrightarrow{\partial}_\mu K + i\, g_3^{\phi KK}\, \phi_{\mu\nu}(\partial_\mu K^+\cdot\partial_\nu K \\
&\qquad - \partial_\nu K^+\cdot\partial_\mu K) \\
&+ i[g_1^{K^*K\pi}\, K^{*+}_\mu \tau K\, \partial_\mu\pi - i g_2^{K^*K\pi}\, K^{*+}_\mu \tau\, \partial_\mu K\cdot\pi \\
&+ g_3^{K^*K\pi}\, K^{*+}_{\mu\nu}\tau(\partial_\mu K\, \partial_\nu\pi - \partial_\nu K\, \partial_\mu\pi) + h.c] \\
&+ \dots\;,
\end{aligned}$$

where ... stands for the couplings which are not used in the calculations and K and K^* stand for the coloumn vector, $\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ etc.

The various coupling constants are,

$$g_1^{\rho\pi\pi} = g\, L_\pi\, Z_\pi\, Z_\rho^{1/2} \quad ,$$
$$g_3^{\rho\pi\pi} = g\, (\xi_\pi^2\, h_4 - \delta'\, L_\pi^2)\, Z_\pi\, Z_\rho^{1/2}/2 \quad ,$$
$$g_1^{\rho KK} = g\, L_K\, Z_K\, Z_\rho^{1/2}/2 \quad ,$$
$$g_3^{\rho KK} = g\, (\xi_K^2\, h_4 - \delta'\, L_K^2)\, Z_K\, Z_\rho^{1/2}/4 \quad ,$$
$$g_1^{\phi KK} = \sqrt{3}\, g\, L_K\, Z_K\, Z_\phi^{1/2}/2 \quad ,$$
$$g_3^{\phi KK} = \sqrt{3}\, g(\xi_K^2\, h_5 - \delta'\, L_K^2)\, Z_K\, Z_\phi^{1/2}/4 \quad ,$$
$$g_1^{K^*K\pi} = g\, (L_\pi - \sqrt{3}\, g f_8\, \xi_\pi/2)\, Z_1/2 \quad ,$$
$$g_2^{K^*K\pi} = g\, (L_K + \sqrt{3}\, g f_8\, \xi_K/2)\, Z_1/2 \quad ,$$
$$g_3^{K^*K\pi} = g\, (\xi_\pi \xi_K\, h_6 - \delta'\, L_\pi\, L_K)\, Z_1/4 \quad ,$$

where,

$$Z_1 = (Z_\pi\, Z_K\, Z_{K^*})^{1/2} \quad ..$$

2. <u>AVP Vertex</u>:

$$\begin{aligned}
L(AVP) = {} & \frac{ig^2}{2}\Big\{ [\, \hat{Z}_\mu,\, f\,]_+\, [\, \hat{Y}_\mu, \Pi]_- - [\, \hat{Z}_\mu, \Pi]_+ [\, \hat{Y}_\mu,\, f]_- \Big\} \\
& - \frac{ig}{4\sqrt{2}}\Big\{ [\, \hat{Y}_{\mu\nu},\, P^A_{\mu\nu}]_+\, (1 + h_1 f^2) \\
& + 2h_2\, \hat{Y}_{\mu\nu}\, f\, P^A_{\mu\nu}\, f\, \Big\} \\
& - \frac{ihg}{4}\Big\{ \hat{Y}_{\mu\nu}\, ([\, [\hat{Z}_\mu,\, f]_+,\, P_\nu]_- + [P_\mu,\, [\hat{Z}_\nu,\, f]_+]_-) \\
& \qquad - \hat{Y}_{\mu\nu}\, ([P_\mu,\, [\hat{Y}_\nu,\, f]_-\,]_+ - [P_\nu,\,][\hat{Y}_\mu,\, f]_-]_+) \Big\} \\
& - \frac{ig}{4\sqrt{2}}\Big\{ [\, \hat{Z}_{\mu\nu},\, P^V_{\mu\nu}\,]_+\, (\, 1 + h_1\, f^2) \\
& \qquad - 2h_2\, \hat{Z}_{\mu\nu}\, f\, P^V_{\mu\nu}\, f \Big\} + \frac{ih_1}{4}\Big\{ [\, \hat{Z}_{\mu\nu},\, \hat{Z}_{\mu\nu}\,]_+ [\Pi,\, f]_- \Big\} \\
& - \frac{ih_2}{2}\Big\{ \hat{Z}_{\mu\nu}(f\, \hat{Y}_{\mu\nu}\Pi - \Pi\hat{Y}_{\mu\nu}\, f) \Big\} \, , \qquad \text{(A.10)}
\end{aligned}$$

where,

$$P^{A}_{\mu\nu} = [\Pi_\mu, \hat{Z}_\nu]_- + [\hat{Z}_\mu, \Pi_\nu]_- ,$$

$$P^{V}_{\mu\nu} = [\hat{Y}_\mu, \Pi_\nu]_- - [\hat{Y}_\nu, \Pi_\mu]_- .$$

The isospin decomposition of (10) is as follows,

$$\begin{aligned}
L(AVP) = & \Big[g_1^{A_1\rho\pi} A_{1\mu}\pi \times \rho_\mu + g_2^{A_1\rho\pi} (A_{1\mu}\partial_\nu\pi - A_{1\nu}\partial_\mu\pi) \times \rho_{\mu\nu} \\
& + g_3^{A_1\rho\pi} A_{1\mu\nu}\cdot(\partial_\nu\pi \times \rho_\mu - \partial_\mu\pi \times \rho_\nu) \\
& + g_4^{A_1\rho\pi} A_{1\mu\nu}\pi \times \rho_{\mu\nu} \Big] \\
& + i\Big[g_1^{K_A\rho K} K^+_{A\mu}\tau K \rho_\mu + g_2^{K_A\rho K} (K^+_{A\mu}\tau\partial_\nu K - K^+_{A\nu}\tau\partial_\mu K)\cdot\rho_{\mu\nu} \\
& + g_3^{K_A\rho K} K^+_{A\mu\nu}\tau(\partial_\nu K\rho_\mu - \partial_\mu K\rho_\nu) \\
& + g_4^{K_A\rho K} K^+_{A\mu\nu}\tau K\rho_{\mu\nu} + h.c \Big] . \\
& + i\Big[g_1^{K_A K^*\pi} K^+_{A\mu}\tau K^*_\mu\cdot\pi + \\
& + g_2^{K_A K^*\pi} (K^+_{A\mu}\partial_\nu\pi - K^+_{A\nu}\partial_\mu\pi)\tau K^*_{\mu\nu} \\
& + g_3^{K_A K^*\pi} K^+_{A\mu\nu}\tau(\partial_\nu\pi\, K^*_\mu - \partial_\mu\pi\, K^*_\nu) \\
& + g_4^{K_A K^*\pi} K^+_{A\mu\nu}\tau(\partial_\mu K^*_\nu - \partial_\nu K^*_\mu)\pi + h.c \Big] \\
& + i\Big[g_1^{EK^*K} E_\mu K^+ . K^*_\mu + g_2^{EK^*K}(E_\mu\partial_\nu K^+ - E_\nu\partial_\mu K^+) . K^*_{\mu\nu} \\
& + g_3^{EK^*K} E_{\mu\nu}(\partial_\nu K^+ . K^*_\mu - \partial_\mu K^+ . K^*_\nu) \\
& + g_4^{EK^*K} E_{\mu\nu} K^+ . K^*_{\mu\nu} + h.c \Big] \\
& + \ldots\ldots .
\end{aligned}$$

The coupling constants are,

$$g_1^{A_1\rho\pi} = \sqrt{2}\ g^2 c\ Z_2 \ ,$$

$$g_2^{A_1\rho\pi} = g\ (\xi_\pi\ h_4 + \sqrt{2}\ h\ L_\pi\ c)\ Z_2/2 \ ,$$

$$g_3^{A_1\rho\pi} = -g\ \xi_\pi\ h_7\ Z_2/2 \ ,$$

$$g_4^{A_1\rho\pi} = c\ h_2\ Z_2/\sqrt{2} \ ,$$

$$g_1^{K_A\rho K} = g^2\ (\ c + d\ )\ Z_3/2\sqrt{2} \ ,$$

$$g_2^{K_A\rho K} = g\ (\xi_K\ h_4 + h\ (c + d)\ L_K/\sqrt{2})\ Z_3/4 \ ,$$

$$g_3^{K_A\rho K} = -g\,\xi_K\ h_7\ Z_3/4 \ ,$$

$$g_4^{K_A\rho K} = -\ (h_1\ (c - d)/2 + h_2\ c)\ Z_3/2\sqrt{2} \ ,$$

$$g_1^{K_A K^*\pi} = -g^2\ d\ Z_4/\sqrt{2} \ ,$$

$$g_2^{K_A K^*\pi} = -g\ (\xi_\pi\ h_6 + h\ L_\pi\ (c + d)/\sqrt{2})\ Z_4/4 \ ,$$

$$g_3^{K_A K^*\pi} = g\ (\xi_\pi\ h_8 + h\ L_\pi\ (c - d)/\sqrt{2})\ Z_4/4 \ ,$$

$$g_4^{K_A K^*\pi} = d\ h_2\ Z_4/2\sqrt{2} \ ,$$

$$g_1^{EK^*K} = \sqrt{3/2}\ g^2\ d\ Z_5 \ ,$$

$$g_2^{EK^*K} = \sqrt{3}\ g\ (\xi_K\ h_6 + \sqrt{2}\ h\ L_K\ (c + 2d)/3)\ Z_5/4 \ ,$$

$$g_3^{EK^*K} = -\sqrt{3}\ g\ (\xi_K\ h_8 + h\ L_K\ (c - d)/3\sqrt{2})\ Z_5/4 \ ,$$

$$g_4^{EK^*K} = -\ (h_1\ (c - d)/2 + h_2\ (c + 2d)\ Z_5/2\sqrt{6} \ ,$$

$$Z_2 = (Z_\pi\ Z_\rho\ Z_{A_1})^{1/2} \ , \quad Z_3 = (Z_K\ Z_\rho\ Z_{K_A})^{1/2} \ ,$$

$$Z_4 = (Z_\pi\ Z_{K^*}\ Z_{K_A})^{1/2} \ , \quad Z_5 = (Z_K\ Z_{K^*}\ Z_E)^{1/2} \ .$$

3. APS Vertex:

$$L\,(APS) = \frac{g}{\sqrt{2}}\Big\{P_\mu\,[\hat{Z}_\mu, \Phi]_+ - S_\mu\,[\hat{Z}_\mu, \Pi]_+ + \frac{g}{\sqrt{2}}\left(-[\Pi_\mu, \Phi]_+ + i[\Phi_\mu, \Pi]_-\right)[\hat{Z}_\mu, f]_+\Big\}$$
$$- \frac{ig}{4\sqrt{2}}\Big\{[\hat{Z}_{\mu\nu}, R_{\mu\nu}]_+(1 + h_1 f^2) - 2h_2\,\hat{Z}_{\mu\nu}\,f\,R_{\mu\nu}\,f\Big\}$$
$$+ \frac{h}{\sqrt{2}}\Big\{\hat{Z}_{\mu\nu}\left([S_\mu, P_\nu]_+ - [S_\mu, P_\nu]_+\right)\Big\}, \qquad (A.11)$$

where,

$$R_{\mu\nu} = [\Phi_\mu, \Pi_\nu]_- - [\Phi_\nu, \Pi_\mu]_- \;.$$

The isospin decomposition of (11) is,

$$\begin{aligned}
L\,(APS) = {} & \Big[\, g_1^{A_1\pi s_8}\, A_{1\mu}\cdot\partial_\mu\pi s_8 - g_2^{A_1\pi s_8}\, A_{1\mu}\cdot\pi\partial_\mu s_8 \\
& + g_3^{A_1\pi s_8}\, A_{1\mu\nu}(\partial_\mu\pi\partial_\nu s_8 - \partial_\nu\pi\,\partial_\mu s_8) + s_8 \leftrightarrow s_0 \,\Big] \\
& + \Big[\, g_1^{K_A\pi S_K}\, K^+_{A\mu}\tau S_K\,\partial_\mu\pi - g_2^{K_A\pi S_K}\, K^+_{A\mu}\tau\partial_\mu S_K\,\pi \\
& + g_3^{K_A\pi S_K}\, K^+_{A\mu\nu}\tau(\partial_\mu\pi\,\partial_\nu S_K - \partial_\nu\pi\,\partial_\mu S_K)\Big] \\
& + \Big[\, g_1^{K_A K s_8}\, K^+_{A\mu}\cdot\partial_\mu K s_8 - g_2^{K_A K s_8}\, K^+_{A\mu}\cdot K\,\partial_\mu s_8 \\
& + g_3^{K_A K s_8}\, K^+_{A\mu\nu}\cdot(\partial_\mu K\,\partial_\nu s_8 - \partial_\nu K\cdot\partial_\mu s_8) + s_8 \leftrightarrow s_0\Big] \\
& + \Big[\, g_1^{E\pi S_\pi}\, E_\mu\,\partial_\mu\pi S_\pi - g_2^{E\pi S_\pi}\, E_\mu\,\pi\cdot\partial_\mu S_\pi \\
& + g_3^{E\pi S_\pi}\, E_{\mu\nu}(\partial_\mu\pi\cdot\partial_\nu S_\pi - \partial_\nu\pi\cdot\partial_\mu S_\pi)\Big] \\
& + \Big[\, g_1^{E\pi_8 s_8}\, E_\mu\partial_\mu\pi_8 s_8 - g_2^{E\pi_8 s_8}\, E_\mu\,\pi_8\partial_\mu s_8 \\
& + g_3^{E\pi_8 s_8}\, E_{\mu\nu}(\partial_\mu\pi_8\partial_\nu s_8 - \partial_\nu\pi_8\partial_\mu s_8) \\
& \qquad + \pi_8 \leftrightarrow \pi_0 + s_8 \leftrightarrow s_0\,\Big], \\
& + \;\ldots\;,
\end{aligned}$$

where,

$$g_1^{A_1\pi s_8} = g_1^{A_1\pi s_0}/\sqrt{2} = g(1-2\sqrt{2}\, c\xi_\pi)\, Z_6/\sqrt{3} \ ,$$

$$g_2^{A_1\pi s_8} = g_2^{A_1\pi s_0}/\sqrt{2} = g\, Z_6/\sqrt{3} \ ,$$

$$g_3^{A_1\pi s_8} = g_3^{A_1\pi s_0}/\sqrt{2} = h\, L_\pi\, Z_6/\sqrt{3} \ ,$$

$$g_1^{K_A\pi S_K} = g(1 - 2\sqrt{2}\, c\xi_\pi)\, Z_7/2 \ ,$$

$$g_2^{K_A\pi S_K} = g(L_S + (c + d)\xi_S/\sqrt{2})\, Z_7/2 \ ,$$

$$g_3^{K_A\pi S_K} = (h\, L_\pi\, L_S - g\, \xi_S\, \xi_\pi\, h_8)\, Z_7/4 \ ,$$

$$g_1^{K_A K s_8} = -g_1^{K_A K s_0}/2\sqrt{2} = -\ g(1 - \sqrt{2}g(c + d)\xi_K)\, Z_8/2\sqrt{3} \ ,$$

$$g_2^{K_A K s_8} = -g_2^{K_A K s_0}/2\sqrt{2} = -\ g\, Z_8/2\sqrt{3} \ ,$$

$$g_3^{K_A K s_8} = -g_3^{K_A K s_0}/2\sqrt{2} = -\ h\, L_K\, Z_8/2\sqrt{3} \ ,$$

$$g_1^{E\pi S_\pi} = g(1 - 2\sqrt{2}\, c\xi_\pi)\, Z_9/\sqrt{3} \ ,$$

$$g_2^{E\pi S_\pi} = g\, Z_9/\sqrt{3} \ ,$$

$$g_3^{E\pi S_\pi} = h\, L_\pi\, Z_9/2\sqrt{3} \ ,$$

$$g_1^{EKS_K} = -g\, (2L_K - 1)\, Z_{10}/2\sqrt{3} \ ,$$

$$g_2^{EKS_K} = -g\, (L_S - \sqrt{2}\, (c + 2d)\, \xi_S)\, Z_{10}/2\sqrt{3} \ ,$$

$$g_3^{EKS_K} = -\ (h\, L_K\, L_S - 3g\xi_S\xi_K\, h_8)\, Z_{10}/4\sqrt{3} \ ,$$

$$g_1^{E\pi_8 s_8}/g = g_3^{E\pi_8 s_8}/h = (L_{81} - 4L_{82})\, Z_E^{1/2}/3\sqrt{3} \ ,$$

$$g_1^{E\pi_8 s_0}/g = g_3^{E\pi_8 s_0}/h = \sqrt{2}\, (L_{81} + 2L_{82})\, Z_E^{1/2}/3\sqrt{3} \ ,$$

$$g_1^{E\pi_0 s_0}/g = g_3^{E\pi_0 s_0}/h = -2g\,(L_{01} - L_{02})\, Z_E^{1/2}/3\sqrt{3} \quad ,$$

$$g_1^{E\pi_0 s_8}/g = g_3^{E\pi_0 s_8}/h = \sqrt{2}\,(L_{01} - 2L_{02})\, Z_E^{1/2}/3\sqrt{3} \quad ,$$

$$\sqrt{2}\, g_2^{E\pi_8 s_8} = -g_2^{E\pi_8 s_0} = g_2^{E\pi_0 s_8} = -\sqrt{2/3}\, g\, Z_E^{1/2} \quad ,$$

$$g_2^{E\pi_0 s_0} = 0 \quad ,$$

where,

$$Z_6 = (Z_\pi\, Z_{A_1})^{1/2} \quad ,$$

$$Z_7 = (Z_{K_A}\, Z_\pi\, Z_{S_K})^{1/2} \quad ,$$

$$Z_8 = (Z_{K_A}\, Z_K)^{1/2} \quad ,$$

$$Z_9 = (Z_E\, Z_\pi)^{1/2} \quad ,$$

$$Z_{10} = (Z_E\, Z_K\, Z_{S_K})^{1/2} \quad .$$

4. <u>SPP Vertex:</u>

$$\begin{aligned}
L(SPP) = {} & \frac{g}{\sqrt{2}}\left\{ iP_\mu\,[\Phi_\mu, \Pi]_- - S_\mu\,[\Pi_\mu, \Pi]_+ + P_\mu\,[\Pi_\mu, \Phi]_+ \right\} \\
& - \frac{\alpha}{2}\,[\, -6\{\Phi\Pi^2\} + 6\{\Phi\Pi\}\{\Pi\} + 3\{\Phi\}\{\Pi^2\} \\
& \qquad - 3\{\Phi\}\{\Pi\}^2\,] \\
& - 2\beta\left\{ (\Phi f + f\Phi)\,\Pi^2 - \Pi f \Pi \right\} - 2\chi\{\Pi\}^2\,\{\Phi f\} \\
= {} & [\, g_1^{S_\pi KK}\, \partial_\mu S_\pi\, \partial_\mu K^+ \tau K + g_3^{S_\pi KK}\, S_\pi \cdot \partial_\mu K^+ \tau\, \partial_\mu K \\
& \qquad + g_4^{S_\pi KK}\, S_\pi\, K^+ \tau\, K\,] \\
& + [\, g_1^{S_\pi \pi\pi_8}\, \partial_\mu S_\pi \cdot \partial_\mu \pi\pi_8 + g_2^{S_\pi \pi\pi_8}\, \partial_\mu S_\pi \cdot \pi\,\partial_\mu \pi_8
\end{aligned}$$

$$+ g_3^{S_\pi \pi\pi_8} S_\pi \cdot \partial_\mu \pi \, \partial_\mu \pi_8 + g_4^{S_\pi \pi\pi_8} S_\pi \cdot \pi \pi_8 + \pi_0 \leftrightarrow \pi_8]$$

$$+ [g_1^{S_K K\pi} \partial_\mu S_K^+ \tau \partial_\mu K \cdot \pi + g_2^{S_K K\pi} \partial_\mu S_K^+ \tau K \cdot \partial_\mu \pi$$

$$+ g_3^{S_K K\pi} S_K^+ \tau \, \partial_\mu K \cdot \partial_\mu \pi + g_4^{S_K K\pi} S_K^+ \tau K \cdot \pi + h.c]$$

$$+ [g_1^{s_8 KK} \partial_\mu s_8 \, \partial_\mu K^+ \cdot K + g_3^{s_8 KK} s_8 \partial_\mu K^+ \cdot \partial_\mu K$$

$$+ g_4^{s_8 KK} s_8 \, K^+ \cdot K + s_0 \leftrightarrow s_8]$$

$$+ [g_1^{s_8 \pi\pi} \partial_\mu s_8 \, \partial_\mu \pi \cdot \pi + g_3^{s_8 \pi\pi} s_8 \, \partial_\mu \pi \cdot \partial_\mu \pi$$

$$+ g_4^{s_8 \pi\pi} s_8 \pi \cdot \pi + s_0 \leftrightarrow s_8]$$

$$+ [g_1^{s_8 \pi_8 \pi_8} \partial_\mu s_8 \, \partial_\mu \pi_8 \pi_8 + g_3^{s_8 \pi_8 \pi_8} s_8 \, \partial_\mu \pi_8 \partial_\mu \pi_8$$

$$+ g_4^{s_8 \pi_8 \pi_8} s_8 \pi_8 \pi_8 + s_8 \leftrightarrow s_0 + \pi_8 \leftrightarrow \pi_0]$$

$$+ [g_1^{s_8 \pi_8 \pi_0} \partial_\mu s_8 \, \partial_\mu \pi_8 \pi_0 + g_2^{s_8 \pi_8 \pi_0} \partial_\mu s_8 \, \partial_\mu \pi_0$$

$$+ g_3^{s_8 \pi_8 \pi_0} s_8 \, \partial_\mu \pi_8 \, \partial_\mu \pi_0 + g_4^{s_8 \pi_8 \pi_0} s_8 \pi_8 \pi_0 + s_8 \leftrightarrow s_0]$$

$$+ \ldots , \qquad \text{(A.12)}$$

where,

$$L_K \, g_1^{S_\pi KK} = - g_3^{S_\pi KK} = \frac{g}{2} L_K \xi_K Z_K ,$$

$$g_4^{S_\pi KK} = (3\alpha/\sqrt{2} - \beta(f_0 + 4 f_8/\sqrt{3})) \, Z_K ,$$

$$L_\pi \, g_1^{s_8 \pi\pi} = - g_3^{s_8 \pi\pi} = - \frac{g}{\sqrt{3}} L_\pi \xi_\pi \, Z_\pi ,$$

$$L_\pi \, g_1^{s_0 \pi\pi} = - g_3^{s_0 \pi\pi} = - \sqrt{2/3} \, L_\pi \xi_\pi \, Z_\pi ,$$

$$g_4^{s_8\pi\pi} = (\sqrt{3}/2\ \alpha - \sqrt{2/3}\ \beta c - 2\gamma f_8)\ Z_\pi \ ,$$

$$g_4^{s_0\pi\pi} = -(\sqrt{3}\alpha/2 + 2\ \beta c/\sqrt{3} + \sqrt{6}\gamma f_0)\ Z_\pi \ ,$$

$$\sqrt{2}\ g_1^{S_\pi\pi\pi_8} = g_1^{S_\pi\pi\pi_0} = -\ g\xi_\pi\ Z_\pi^{1/2}/\sqrt{6} \ ,$$

$$g_2^{S_\pi\pi\pi_8}/\xi_{88} = g_2^{S_\pi\pi\pi_0}/\xi_{80} = -\ g\ Z_\pi^{1/2}/2\sqrt{3} \ ,$$

$$g_3^{S_\pi\pi\pi_8} = g\ (\xi_\pi\ L_{81} + L_\pi\ \xi_{88})\ Z_\pi^{1/2}/2\sqrt{3} \ ,$$

$$g_3^{S_\pi\pi\pi_0} = g\ (\xi_\pi\ L_{01} + L_\pi\ \xi_{80})\ Z_\pi^{1/2} \ ,$$

$$g_4^{S_\pi\pi\pi_8} = (\sqrt{6}\alpha - 2\sqrt{2/3}\ \beta c)\ Z_\pi^{1/2} \ ,$$

$$g_4^{S_\pi\pi\pi_0} = (\sqrt{3}\alpha + 2\beta c/\sqrt{3})\ Z_\pi^{1/2} \ ,$$

$$g_1^{S_K K\pi} = -\ g(L_S\xi_K - \xi_S\ L_K)\ Z_{11}/2 \ ,$$

$$g_2^{S_K K\pi} = -\ g(L_S\xi_\pi + \xi_S\ L_\pi)\ Z_{11}/2 \ ,$$

$$g_3^{S_K K\pi} = g(L_K\xi_\pi + L_\pi\ \xi_K)\ Z_{11}/2 \ ,$$

$$g_4^{S_K K\pi} = (3\alpha/\sqrt{2} - \sqrt{2}\beta\ d)\ Z_{11}$$

$$= (\sqrt{2/3}\ b_0 - f_0 b_8/f_8)\ Z_{11}/(2f_0^2 + f_0 f_8/\sqrt{3} - f_8^2/3) \ ,$$

$$L_K\ g_1^{s_8KK} = -g_3^{s_8KK} = g\ L_K\xi_K\ Z_K/2\sqrt{3} \ ,$$

$$g_4^{s_8KK} = (-\sqrt{3}/2\ \alpha + \beta(f_0 - 14f_8/\sqrt{3})/\sqrt{3} - 4\gamma f_8)\ Z_K \ ,$$

$$L_K\ g_1^{s_0KK} = -g_3^{s_0KK} = -\sqrt{2/3}\ g\ L_K\xi_K\ Z_K \ ,$$

$$g_4^{s_0KK} = (-\sqrt{3}\alpha - \sqrt{2/3}\ \beta\ (2f_0 - f_8/\sqrt{3}) - 2\sqrt{6}f_0\gamma)\ Z_K \ ,$$

$$g_1^{s_8\pi_8\pi_8} = -3g_3^{s_8\pi_8\pi_8}/(L_{81} - 4L_{82}) = -g\xi_{88}/\sqrt{3} ,$$

$$g_4^{s_8\pi_8\pi_8} = -\sqrt{3}/2\ \alpha + 2c\beta/\sqrt{3} - 2\sqrt{3}f_8\gamma ,$$

$$g_1^{s_0\pi_8\pi_8} = -3g_3^{s_0\pi_8\pi_8}/(L_{81} + 2L_{82}) = -\sqrt{2}g\xi_{88}/\sqrt{3} ,$$

$$g_4^{s_0\pi_8\pi_8} = \sqrt{3}\alpha/2 - 4c\beta/\sqrt{3} ,$$

$$g_1^{s_8\pi_0\pi_0} = -3g_3^{s_8\pi_0\pi_0}/(L_{01} + 2L_{02}) = -\sqrt{2}g\xi_{80}/\sqrt{3} ,$$

$$g_4^{s_8\pi_0\pi_0} = g_1^{s_0\pi_0\pi_0} = 0 ,$$

$$g_3^{s_0\pi_0\pi_0} = 2g(L_{01} - L_{02})\xi_{80}/3\sqrt{3} ,$$

$$g_4^{s_0\pi_0\pi_0} = \sqrt{3}\alpha - 4c\beta/\sqrt{3} - \sqrt{6}f_0\gamma ,$$

$$g_1^{s_8\pi_8\pi_0} = -\sqrt{2/3}\ g\xi_{88} , \quad g_2^{s_8\pi_8\pi_0} = g\xi_{80}/\sqrt{3} ,$$

$$g_3^{s_8\pi_8\pi_0} = g(\xi_{80}(L_{81} - 4L_{82}) + \sqrt{2}\ \xi_{88}(L_{01} + 2L_{02}))/3\sqrt{3} ,$$

$$g_4^{s_8\pi_8\pi_0} = -5\sqrt{3}\ \alpha + 4c\beta/\sqrt{3} ,$$

$$g_{1,4}^{s_0\pi_8\pi_0} = 0 , \quad g_2^{s_0\pi_8\pi_0} = -\sqrt{2/3}\ g\xi_{80} ,$$

$$g_3^{s_0\pi_8\pi_0} = \sqrt{2}g\ (\xi_{80}(L_{81} + 2L_{82}) + \sqrt{2}\ (L_{01} - L_{02})\xi_{88})/3\sqrt{3} ,$$

where,

$$Z_{11} = (Z_\pi\ Z_K\ Z_{S_K})^{1/2} .$$

5. PPPP Vertex:

$$
\begin{aligned}
L(PPPP) = &- \frac{g^2}{4}\left\{[\Pi_\mu, \Pi]_+ [\Pi_\mu, \Pi]_+\right\} - \frac{1}{2}\left[\beta\{\Pi^4\} + \gamma\{\Pi^2\}^2\right] \\
&- \frac{gh}{4}\left\{[\Pi_\mu, \Pi_\nu]_- \cdot [P_\mu, P_\nu]_-\right\} \\
&+ \frac{g^2}{8}\left\{[\Pi_\mu, \Pi_\nu]_- [\Pi_\mu, \Pi_\nu]_- (1 + h_1 f^2)\right. \\
&\qquad \left. + h_2 [\Pi_\mu, \Pi_\nu]_- f [\Pi_\mu, \Pi_\nu]_- f\right\} \\
= &\left[g_1^{\pi\pi}(\pi.\pi)^2 + g_2^{\pi\pi}(\partial_\mu\pi.\pi)^2\right. \\
&\left. + g_3^{\pi\pi}\, \partial_\mu\pi \times \partial_\nu\pi . \partial_\mu\pi \times \partial_\nu\pi\right] \\
&+\left[g_1^{KK}(K^+.K)^2 + g_2^{KK}\left((\partial_\mu K^+.K + K^+.\partial_\mu K)\, \partial_\mu K^+.K\right.\right. \\
&\qquad \left. + (K^+.\partial_\mu K)^2 + \partial_\mu K^+.\partial_\mu K K^+.K\right) \\
&+ g_3^{KK}\left(\partial_\nu K^+.\partial_\mu K\, \partial_\nu K^+.\partial_\mu K - \partial_\mu K^+.\partial_\mu K\, \partial_\nu K^+.\partial_\nu K\right) \\
&\left. + g_4^{KK}\left(\partial_\mu K^+.\partial_\nu K - \partial_\nu K^+.\partial_\mu K\right)^2\right] \\
&+\left[g_1^{K\pi} K^+.K\pi.\pi + g_2^{K\pi}(\partial_\mu K^+.K + K^+.\partial_\mu K)\, \partial_\mu\pi.\pi\right. \\
&+ g_3^{K\pi} K^+.K\, \partial_\mu\pi.\partial_\mu\pi + g_4^{K\pi}\, \partial_\mu K^+.\partial_\mu K\pi.\pi \\
&+ i g_5^{K\pi}\left(\partial_\mu K^+\tau K - K^+\tau\, \partial_\mu K\right) \partial_\mu\pi \times \pi \\
&+ i g_6^{K\pi}\left(\partial_\nu K^+\tau\, \partial_\mu K - \partial_\mu K^+\tau\, \partial_\nu K\right) \partial_\nu\pi \times \partial_\mu\pi \\
&+ g_7^{K\pi}\left(-2\partial_\mu\pi.\partial_\mu\pi\, \partial_\nu K^+.\partial_\nu K +\right. \\
&\qquad \left.\left. + \partial_\mu\pi.\partial_\nu\pi(\partial_\mu K^+.\partial_\nu K + \partial_\nu K^+.\partial_\mu K)\right)\right] \\
&+\left[g_1 (\xi_{88}\, \partial_\mu\pi_8 + \xi_{80}\, \partial_\mu\pi_0)^2\, \pi.\pi\right. \\
&+ g_2 (\xi_{88}\, \partial_\mu\pi_8 + \xi_{80}\, \partial_\mu\pi_0)(\pi_8/\sqrt{6} + \pi_0/\sqrt{3})\partial_\mu\pi.\pi
\end{aligned}
$$

$$+ g_3\, \partial_\mu\pi\cdot\partial_\mu\pi\, (\pi_8/\sqrt{6} + \pi_0/\sqrt{3})^2$$
$$+ g_4\, (\pi_8/\sqrt{6} + \pi_0/\sqrt{2})^2\pi\cdot\pi + g_5(\pi_8\pi_8 + \pi_0\pi_0)\pi\cdot\pi] \;,$$
$$+ \ldots \quad . \tag{A.13}$$

The coupling constants are,

$$g_1^{\pi\pi} = -(\beta + 2\gamma)\, Z_\pi^2/4 \;,$$
$$g_2^{\pi\pi} = -g^2 \xi_{\varphi\pi}^2\, Z_\pi^2/2 \;,$$
$$g_3^{\pi\pi} = g^2 \xi_{\varphi\pi}^2 (\, h_4 \xi_{\varphi\pi}^2 + 2\delta'\, L_\pi^2\,)\, Z_\pi^2/4 \;,$$
$$g_1^{KK} = -(\beta + 2\gamma)\, Z_K^2/2 \;,$$
$$g_2^{KK} = -g^2 \xi_{\varphi K}^2\, Z_K^2/2 \;,$$
$$g_3^{KK} = g^2 \xi_{\varphi K}^2 (\xi_{\varphi K}^2\, h_4 - 2\delta' L_K^2)\, Z_K^2/4 \;,$$
$$g_1^{KK} = g^2 \xi_{\varphi K}^2 (\xi_{\varphi K}^2 (1+(h_1+h_2)d^2) - 2\delta'\, L_K^2)\, Z_K^2/4 \;,$$
$$g_1^{K\pi} = -(\beta + 2\gamma)\, Z_\pi\, Z_K \;,$$
$$g_2^{K\pi} = -3g_5^{K\pi} = -3g^2 \xi_{\varphi\pi} \xi_{\varphi K}\, Z_\pi\, Z_K/8$$
$$g_3^{K\pi}/\xi_{\varphi\pi}^2 = g_4^{K\pi}/\xi_{\varphi K}^2 = -g^2\, Z_\pi\, Z_K/8$$
$$g_6^{K\pi} = g^2 [\xi_{\varphi\pi}^2 \xi_{\varphi K}^2\, (3(1+h_1c^2) + c(2c+d)h_2)$$
$$+ 2\delta'(\xi_{\varphi\pi}^2\, L_K^2 + \xi_{\varphi K}^2\, L_\pi^2 + \xi_{\varphi K}\xi_{\varphi\pi}\, L_K L_\pi)]\, Z_\pi Z_K/8 \;,$$
$$g_7^{K\pi} = g^2 \xi_{\varphi\pi}\xi_{\varphi K}\, [\xi_{\varphi\pi}\xi_{\varphi K}(1+h_1c^2+h_2cd) + 2\delta' L_\pi L_K]\, Z_\pi Z_K/8 \;,$$
$$g_1 = -g^2\, Z_\pi/6 \;,$$

$$g_2 = -2\sqrt{2/3}\, g^2 \xi_\pi Z_\pi \quad ,$$

$$g_3 = -g^2 \xi_\pi^2 Z_\pi \quad ,$$

$$g_4 = -3\beta Z_\pi \quad ,$$

$$g_5 = -\gamma Z_\pi \quad ,$$

The $\eta\eta'\pi\pi$ in (13) is as follows,

$$L(\eta'\eta\pi\pi) = g_1^{\eta\pi}\, \partial_\mu\eta'\, \partial_\mu\eta\, \pi\cdot\pi + g_2^{\eta\pi}\, \partial_\mu\eta'\,\eta\,\partial_\mu\pi\cdot\pi$$
$$+ g_3^{\eta\pi}\, \eta'\, \partial_\mu\eta\, \partial_\mu\pi\cdot\pi + g_4^{\eta\pi}\, \eta'\eta\, \partial_\mu\pi\cdot\partial_\mu\pi$$
$$+ g_5^{\eta\pi}\, \eta'\eta\, \pi\cdot\pi \quad .$$

where,

$$g_1^{\eta\pi} = 2g_1\, [\xi_{88}^2\, c_{8\eta}c_{8\eta'} + \xi_{80}^2\, c_{0\eta}\, c_{0\eta'}$$
$$+ \xi_{88}\xi_{80}\, (c_{8\eta}\, c_{0\eta'} + c_{8\eta'}\, c_{0\eta})] \quad ,$$

$$g_2^{\eta\pi} = g_2\, (\xi_{88}\, c_{8\eta'} + \xi_{80}\, c_{0\eta'})\, (c_{8\eta}/\sqrt{6} + c_{0\eta}/\sqrt{3}) \quad ,$$

$$g_3^{\eta\pi} = g_2\, (\xi_{88}\, c_{8\eta} + \xi_{80}\, c_{0\eta})\, (c_{8\eta'}/\sqrt{6} + c_{0\eta'}/\sqrt{3}) \quad ,$$

$$g_4^{\eta\pi} = g_3\, [c_{8\eta}\, c_{8\eta'} + 2c_{0\eta}c_{0\eta'} + \sqrt{2}(c_{8\eta}c_{0\eta'}$$
$$+ c_{8\eta'}\, c_{0\eta})]/3 \quad ,$$

$$g_5^{\eta\pi} = g_4^{\eta\pi}\, g_4/g_3 + 2g_5\, (c_{8\eta}\, c_{8\eta'} + c_{0\eta}\, c_{0\eta'}) \quad .$$

6. APPP Vertex:

$$
\begin{aligned}
L(APPP) = & -\frac{g^2}{2}\{[\hat{Z}_\mu, \Pi]_+ [\Pi_\mu, \Pi]_+\} \\
& + \frac{g^2}{2}\{[\hat{Z}_\mu, \Pi_\nu]_- [\Pi_\mu, \Pi_\nu]_- (1 + h_1 f^2) \\
& + h_2 [\hat{Z}_\mu, \Pi_\nu]_- f[\Pi_\mu, \Pi_\nu]_- f\} \\
& + \frac{g}{4\sqrt{2}}\{ - h_1 [\hat{Z}_{\mu\nu}, [\Pi_\mu, \Pi_\nu]_-]_+ [\Pi, f]_- \\
& + 2h_2 \hat{Z}_{\mu\nu} (f[\Pi_\mu, \Pi_\nu]_- \Pi - \Pi[\Pi_\mu, \Pi_\nu]_- f)\} \\
& + \frac{gh}{4}\{\hat{Z}_{\mu\nu}([P_\mu, [\Pi_\nu, \Pi]_+]_+ - [P_\nu, [\Pi_\mu, \Pi]_+]_+) \\
& - ([\hat{Z}_\mu, \Pi_\nu]_- + [\Pi_\mu, \hat{Z}_\nu]_-)[P_\mu, P_\nu]_-\} \\
= & [g_1^{A_1\pi\pi\pi} A_{1\mu} \cdot \pi\, \partial_\mu \pi \cdot \pi + g_2^{A_1\pi\pi\pi} A_{1\mu} \times \partial_\nu \pi \cdot \partial_\mu \pi \times \partial_\nu \pi \\
& + g_3^{A_1\pi\pi\pi} A_{1\mu\nu}(\partial_\mu \pi\, \partial_\nu \pi \cdot \pi - \partial_\nu \pi\, \partial_\mu \pi \cdot \pi) \\
& + g_4^{A_1\pi\pi\pi} A_{1\mu\nu} \times \pi \cdot (\partial_\nu \pi \times \partial_\mu \pi)] \\
& + [g_1^{K_A K\pi\pi} K^+_{A\mu} \cdot K\, \partial_\mu \pi \cdot \pi + g_2^{K_A K\pi\pi} K^+_{A\mu} \cdot \partial_\mu K \pi \cdot \pi \\
& + g_3^{K_A K\pi\pi} ((K^+_{A\mu} \cdot \partial_\nu K + K^+_{A\nu} \cdot \partial_\mu K)\, \partial_\mu \pi \cdot \partial_\nu \pi \\
& \qquad - 2K^+_{A\mu} \cdot \partial_\mu K\, \partial_\nu \pi \cdot \partial_\nu \pi) \\
& + i g_4^{K_A K\pi\pi} (K^+_{A\mu} \tau \partial_\nu K - K^+_{A\nu} \tau \partial_\mu K)\, \partial_\nu \pi \times \partial_\mu \pi \\
& + i g_5^{K_A K\pi\pi} K^+_{A\mu} \tau\, \partial_\mu K\, \pi \times \pi \\
& + g_6^{K_A K\pi\pi} K^+_{A\mu\nu} (\partial_\mu K\, \partial_\nu \pi \cdot \pi - \partial_\nu K\, \partial_\mu \pi \cdot \pi) \\
& + g_7^{K_A K\pi\pi} K^+_{A\mu\nu} K(\partial_\mu \pi \cdot \partial_\nu \pi - \partial_\nu \pi \cdot \partial_\mu \pi)
\end{aligned}
$$

$$+ ig_8^{K_A K\pi\pi} \; K^+_{A\mu\nu}\tau(\partial_\nu K.\partial_\mu\pi \times \pi - \partial_\mu K\, \partial_\nu\pi \times \pi)$$

$$+ ig_9^{K_A K\pi\pi} \; K^+_{A\mu\nu}\tau\, K\, \partial_\mu\pi \times \partial_\nu\pi$$

$$+ ig_{10}^{K_A K\pi\pi} \; K^+_{A\mu}\, \tau K\, \pi \times \partial_\mu\pi \,]$$

$$+[\; g_1^{EKK\pi} \; E_\mu\, K^+\tau K.\partial_\mu\pi$$

$$+ g_2^{EKK\pi} \; E_\mu(\partial_\mu K^+\tau\, K + K^+\tau\, \partial_\mu K).\pi$$

$$+ g_3^{EKK\pi} \; E_\mu(-2\partial_\mu\pi\; \partial_\nu K^+\tau\,\partial_\nu K$$

$$+\partial_\nu\pi\; (\partial_\nu K^+\tau\; \partial_\mu K + \partial_\mu K^+\tau\; \partial_\nu K))$$

$$+ g_4^{EKK\pi} \; E_{\mu\nu}(\partial_\mu\pi\; K^+\tau\; \partial_\nu K - \partial_\nu\pi\; K^+\tau\; \partial_\mu K$$

$$+\partial_\mu\pi\; \partial_\nu K^+\tau K - \partial_\nu\pi\; \partial_\mu K^+\tau\, K) \,]$$

$$+[\; g_1^{\pi_8}\; E_\mu\, \partial_\mu\pi_8\; \pi.\pi + g_2^{\pi_8}\; E_\mu\, \pi_8\; \partial_\mu\pi.\pi$$

$$+ g_3^{\pi_8}\; E_{\mu\nu}(\partial_\mu\pi.\pi\; \partial_\nu\pi_8 - \partial_\nu\pi.\pi\; \partial_\mu\pi_8) + \pi_0 \leftrightarrow \pi_8 \,]$$

$$+ \ldots \;\;\;., \qquad\qquad (A.14)$$

where,

$$g_1^{A_1\pi\pi\pi} \;=\; - g^2\, \xi_\pi\, Z_{12} \;,$$

$$g_2^{A_1\pi\pi\pi} \;=\; g^2(\xi_\pi^3\, h_4 - \delta'\, \xi_\pi\, L_\pi^2)\, Z_{12} \;,$$

$$g_3^{A_1\pi\pi\pi} \;=\; hg\, \xi_\pi\, L_\pi\;\; Z_{12}/2 \;,$$

$$g_4^{A_1\pi\pi\pi} \;=\; gh_2\; c\, \xi_\pi^2\;\; Z_{12}/\sqrt{2} \;,$$

$$g_1^{K_A K\pi\pi} \;=\; -3\, g_{10}^{K_A K\pi\pi} \;=\; -\, 3g^2\xi_\pi\;\; Z_{13}/4 \;,$$

$$g_2^{K_A K\pi\pi} = g_5^{K_A K\pi\pi} = -\,g^2 \xi_K \, Z_{13}/4 \quad ,$$

$$g_3^{K_A K\pi\pi} = g^2 \xi_\pi (\xi_\pi \xi_K (1+h_1 c^2 + h_2 cd) - 2\delta' \, L_\pi \, L_K) \, Z_{13}/8 \quad ,$$

$$g_4^{K_A K\pi\pi} = g^2 \xi_K (\xi_\pi^2 \, (3(1+h_1 c^2) + h_2 cd) - 6\delta' L_\pi^2) \, Z_{13}/8 \quad ,$$

$$g_6^{K_A K\pi\pi} = hg \, L_\pi \xi_\pi \; Z_{13}/4 \quad ,$$

$$g_7^{K_A K\pi\pi} = g(h(L_K \xi_\pi - L_\pi \xi_K) + h_1 d \, \xi_\pi \xi_K/\sqrt{2}) \, Z_{13}/4 \quad ,$$

$$g_8^{K_A K\pi\pi} = g \, \xi_K (h \, L_K + \sqrt{2} \, \xi_\pi d h_2) \, Z_{13}/8 \quad ,$$

$$g_9^{K_A K\pi\pi} = g \, \xi_\pi (h L_\pi + \xi_\pi \, h_1 (c-d)/\sqrt{2}) \, Z_{13}/4 \quad ,$$

$$g_1^{EKK\pi}/\xi_\pi = -2 g_2^{EKK\pi}/\xi_K = g^2 \, Z_{14}/2\sqrt{3} \quad ,$$

$$g_3^{EKK\pi} = \sqrt{3} g^2 \xi_K [-\delta' L_\pi L_K + \xi_\pi \xi_K (1+h_1 c^2 + h_2 cd)] \, Z_{14}/4 \quad ,$$

$$g_4^{EKK\pi} = g(h(L_K \, {}_\pi + 2 L_\pi \, {}_K) - h_1 \xi_\pi \xi_K (c-d)/\sqrt{2}$$

$$-\sqrt{2} h_2 \, \xi_\pi \, \xi_K \, (c+2d)) \, Z_{14}/8\sqrt{3} \quad ,$$

$$g_1^{E\pi_8\pi\pi} = g_1^{E\pi_0\pi\pi}/\sqrt{2} = -2g^2 \xi_\pi \, Z_\pi \, Z_E^{1/2}/3 \quad ,$$

$$g_2^{E\pi_8\pi\pi}/\xi_{88} = g_2^{E\pi_0\pi\pi}/\xi_{80} = -\,g^2 \, Z_\pi \, Z_E^{1/2}/3 \quad ,$$

$$g_3^{E\pi_8\pi\pi} = hg \, (\xi_{88} \, L_\pi - \xi_\pi \, L_{81}) \, Z_\pi \, Z_E^{1/2}/\sqrt{6} \quad ,$$

$$g_3^{E\pi_0\pi\pi} = hg \, (L_\pi \xi_{80}/\sqrt{6} - \xi_\pi \, L_{01}/\sqrt{3}) \, Z_\pi \, Z_E^{1/2} \quad ,$$

where,

$$Z_{12} = Z_\pi^{3/2} \, Z_{A_1}^{1/2} \quad , \quad Z_{13} = Z_\pi \, Z_K^{1/2} \, Z_{K_A}^{1/2} \quad ,$$

$$Z_{14} = Z_K \, Z_\pi^{1/2} \, Z_E^{1/2} \quad .$$